

Pareto Improving Fiscal and Monetary Policies: Samuelson in the New Keynesian Model*

Mark Aguiar

Princeton University,
and NBER

Manuel Amador

University of Minnesota,
Federal Reserve Bank of
Minneapolis, and NBER

Cristina Arellano

Federal Reserve Bank
of Minneapolis

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Abstract

This paper explores the normative and positive consequences of government bond issuances in a New Keynesian model with heterogeneous agents, focusing on how the stock of government bonds affects the cross-sectional allocation of resources in the spirit of Samuelson (1958). We characterize the Pareto optimal levels of government bonds and the associated monetary policy adjustments that should accompany Pareto-improving bond issuances. The paper introduces a simple phase diagram to analyze the global equilibrium dynamics of inflation, interest rates, and consumption in response to changes in the stock of government debt. It provides a tractable tool to explore the use of fiscal policy to escape the Effective Lower Bound (ELB) on nominal interest rates and the resolution of the “forward guidance puzzle.” We show how the fiscal policy stance matters for the effectiveness of the latter. A common theme throughout is that following the monetary policy guidance from the standard Ricardian framework leads to excess fluctuations in inflation and consumption.

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1 Introduction

It is well known, going back to at least Samuelson (1958), that the introduction of outside assets can (Pareto) improve the allocation of consumption in models with heterogeneity. The role of government bonds as a safe store of value, in particular, has played an important role in welfare analyses of both overlapping generations (OLG) and Bewley-Huggett-Aiyagari incomplete market economies. We revisit this idea in a tractable OLG version of the standard New Keynesian model and focus on Pareto improvements for the welfare metric. In particular, we analyze the normative and positive consequences of government bond issuances, and how these depend on the monetary policy rule. The main tool of analysis is a phase diagram that allows easy analysis of the economy's response to a variety of shocks, without recourse to local approximations. We show that the standard monetary rule that is derived in the Ricardian setting generates unnecessary fluctuations in consumption, markups, and inflation in response to a potentially welfare-improving bond issuance.

The focus on Pareto improvements complements the explicitly redistributive policies that have been the focus of the recent policy analyses in heterogeneous agent New Keynesian (HANK) models.¹ A few distinguishing characteristics of our approach should be flagged at the outset. The analysis abstracts from the direct link between monetary policy and the government budget constraint studied in the classic paper of Sargent and Wallace (1981) and the more recent work on the fiscal theory of the price level (e.g., Cochrane, 2023). To make the distinction from the latter literature crystal clear, we model the government as issuing real bonds, although this is done for expositional reasons rather than as a necessary component of the analysis. To contrast with work of Sargent and Wallace (1981), we explore a cashless economy with zero seigniorage revenue.

We build on two standard platforms. To generate a link between government debt and the real economy, we break Ricardian Equivalence using the perpetual youth framework of Blanchard (1985) and Yaari (1965), augmented to include endogenous labor supply but without physical capital. In this framework, we embed a textbook New Keynesian (NK) model of nominal rigidities, as in Rotemberg (1982), Galí (2015), and Woodford (2004). This is a combination that has been used several times in the literature, which we review below. It also generates heterogeneity in a tractable manner, providing clean insights that are applicable to richer heterogeneous agents. New Keynesian models now popular in the quantitative literature.

One useful assumption we make is that the individuals supplying labor (workers) save (in

¹Recent examples of optimal policy in HANK models using a utilitarian criteria include Bhandari et al. (2020), Nuno and Thonmas (2021), Dávilla and Schaab (2022), Acharya, Challe, and Dogra (2020), Bilbiie and Ragot (2021), and McKay and Wolf (2022) and Yang (2022). LeGrand, Martin-Baillon, and Ragot (2021) studies optimal policy using a social welfare function with empirically derived weights.

aggregate) in the government bond, while firm owners (entrepreneurs) save in equity. This dichotomy echoes the reality that many households whose primary income consists of labor earnings do not own shares of firms. As a modeling choice, the key implication is that price setters and holders of government bonds value inter-temporal tradeoffs differently, as there may be a wedge between the equilibrium return on government bonds and the return on equity. While stark, this segmentation proves extremely convenient when analyzing dynamics.

In this environment, we aggregate the workers' problem and derive an Euler condition that relates worker consumption growth to real interest rates and the level government bonds relative to worker consumption (as in Blanchard, 1985). Government bonds are wealth for the generations that are alive and they tend to lower aggregate consumption growth because they decrease the consumption of newborns, that have no wealth, relative to those already alive. An increase in government debt, therefore, requires some combination of a shift in interest rates, consumption growth, or the level of consumption to clear the bond market.

In the product market, prices are set in a monopolistically competitive fashion subject to quadratic adjustment costs, generating a standard NK Phillips curve that relates anticipated inflation, current inflation, and the inverse markup, which in equilibrium proportional to worker consumption. Finally, monetary policy is set through a nominal interest rate rule subject to an effective/zero lower bound (ELB). We will study the implications of a standard Taylor rule, where the nominal interest rate is a linear function of inflation deviations from the inflation target, with a slope coefficient strictly greater than one (i.e., the Taylor Principle holds), and an augmented rule where the nominal interest rate also responds to the level of government debt.

Given these equilibrium restrictions, for a given level of government debt, we characterize the economy as a system of two non-linear ordinary differential equations (ODEs). We do so in terms of inflation and aggregate worker consumption. The system of ODEs is amenable to analysis using a simple phase diagram. We study first the dynamics around an unstable steady state, which represents the monetary authority target inflation rate and the flexible price worker consumption. We focus on the welfare implications of government bond issuances and how they depend on monetary policy. Later on, we study the fiscal implications and dynamics around the other steady state, which is a stable Effective Lower Bound (ELB) equilibrium. This steady state features a nominal interest rate at zero and corresponds to the equilibrium studied by Benhabib, Schmitt-Grohé, and Uribe (2001).

With the equilibrium characterization in hand, we turn to whether expanding the stock of government bonds can improve welfare. This question has been the focus of much work using real models in the tradition of Aiyagari (1994) and also for the classic insights of Samuelson (1958) and Balasko and Shell (1980). Building on Aguiar, Amador, and Arellano (2022), we start our normative analysis studying the feasibility of robust Pareto improvements (RPI), policies that induce

changes in prices and taxes that expand the budget sets of all agents. Consistent with Aguiar, Amador, and Arellano (2022), we find that in a real version of our model, RPIs are feasible if the aggregate savings elasticity with respect to the real interest rate is high enough. With price rigidities, however, the feasibility of an RPI also depends on monetary policy. We show that although monetary policy can increase the elasticity of aggregate savings by running an accommodative policy, such a response does not expand the feasibility of an RPI because it necessarily generates inflation and in doing so decreases entrepreneurs' profits. We show that to implement an RPI, monetary policy needs to set policy according to the augmented interest rate rule that responds to government debt, with a positive coefficient designed to deliver the real interest rate in the flexible price economy.² Monetary and fiscal policy must work in a complementary fashion in order to achieve the Pareto improvement.

We also analyze the positive consequences of various fiscal and monetary policies using the phase diagram. In particular, consider a surprise announcement at $t = 0$ of a one-time debt-financed tax cut (or transfer) that will take place in $t' > 0$ periods. This is equivalent to a "helicopter" distribution of government bonds to taxpayers at t' . We show that if t' is not too distant, inflation, both the nominal and real interest rates, and worker consumption will jump up on the announcement if the Taylor rule were not to react in the future to the fiscal expansion. After the initial impact, inflation, interest rates, and consumption will all continue to increase reaching the new steady state at precisely $t = t'$.³ At this new steady state, inflation, interest rates, and worker consumption are higher.

To understand these dynamics, note that a higher real interest rate at t' is necessary to clear the bond market at a larger stock of government debt. That is, in our non-Ricardian environment, there is a positive steady-state relationship between the amount of government bonds held by savers and the equilibrium real interest rate. If the monetary authority were to follow a standard Taylor rule that increases the nominal interest rate more than one-for-one with inflation, then higher inflation corresponds to a larger real interest rate, and vice versa. Hence, the new steady state has higher inflation, and, via the Phillips curve, a higher level of workers' consumption. Along the path to the new steady state, inflation begins to accelerate immediately upon announcement to avoid an anticipated discontinuity in inflation at t' . This trajectory coincides with accelerating workers' consumption, as well. The response of the economy to an anticipated fiscal deficit is therefore an immediate increase in inflation and worker consumption growth to the new steady state.

This assumes that the monetary policy rule remains invariant to fiscal policy, as in the stan-

²That is, for an RPI, monetary policy should track the real interest rate that will prevail without nominal rigidities but in the presence of the fiscal policy change (the appropriate Wicksellian interest rate, Woodford, 2004).

³It is not obvious whether or why the economy must be at the new steady state at t' , but we postpone a detailed discussion of this to the body of the paper.

standard prescription derived using Ricardian models.⁴ We show that if monetary policy instead follows an augmented interest rate rule that responds to the increase in government debt, then the dynamics can be muted. In the case that monetary policy calls for an increase in nominal rates at t' , to the level associated with real rate in the flexible price economy with the expanded debt, there are no dynamics until t' . At this point, the nominal interest rate and the real rate increase simultaneously with no change in inflation or worker consumption. The analysis highlights the importance of adapting policy for non-Ricardian environments. If the central bank were to follow the policy advice obtained from a standard Taylor rule, it would induce unnecessary fluctuations in both consumption and inflation in response to fiscal deficits. In particular, the central bank would appear to be conscientiously fighting the inflation apparently caused by the lax fiscal authority, when in reality the rigidity of the monetary policy rule is equally to blame for the adverse consequences.

While our focus is primarily on how and when government bonds improve allocations in economies with heterogeneity and nominal rigidities, our model also allows for a tractable analysis of two other questions that have been core topics in monetary economics. The first one relates to the forward guidance puzzle of Del Negro, Giannoni, and Patterson (2023) and McKay, Nakamura, and Steinsson (2016). This puzzle stems from the fact that in the standard representative agent New Keynesian model, an anticipated decline in interest rates far in the future has a large impact on current demand. Our phase diagram shows clearly why, in a non-Ricardian environment, there is no such puzzle, providing an additional insight into the resolution of the puzzle proposed by Del Negro, Giannoni, and Patterson (2023) without relying on linear approximations. In a related paper, Farhi and Werning (2019) study a model with incomplete markets and argue that the forward guidance puzzle is not generally resolved. Based on their work, we discuss the difference in assumptions and show how fiscal policy can affect the effectiveness of forward guidance.

Another extension concerns a transitory decline in the discount rate of savers, a common mechanism for generating an exogenous decline in “aggregate demand.” If the ELB does not bind, we show that the economy experiences a decline in inflation and worker consumption on impact, and then a recovery that features rising inflation and potentially non-monotonic dynamics in worker consumption. Again, the intuition can be obtained from bond market equilibrium; the increased desire to save must be accommodated through lower real interest rates, lower consumption, or anticipated consumption growth. From the Taylor rule, lower real interest rates are generated via lower inflation. The Phillips curve then requires a decline in worker consumption.

⁴More precisely, in Ricardian models the long-run real interest rate is pinned down independently of debt, and hence bond issuances do not alter the long-run target nominal interest rate. This is separate from secular shifts in preferences (discount factors), openness, or demographic changes that may play a role in long-run trends in the real interest rate.

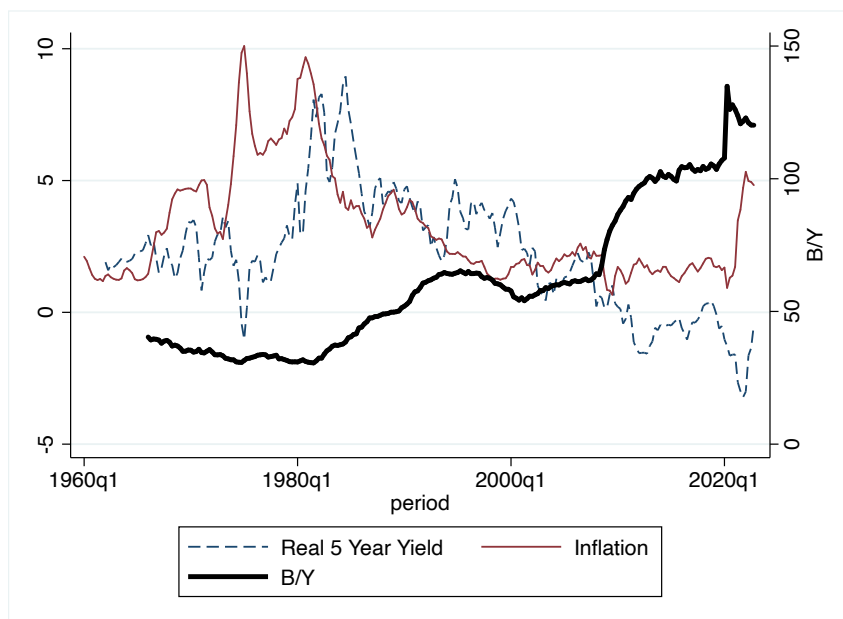


Figure 1: Public Debt, Inflation, and Real Rates in the U.S.

It may be the case that the increase in patience is so severe that the ELB binds and the economy enters a *liquidity trap*. In this case, we show that fiscal policy provides an alternative to the traditional “forward guidance” of Krugman (1998), Eggertsson and Woodford (2003), and Werning (2007). In particular, the fiscal authority can reflate the economy by issuing bonds and rebating the proceeds back to taxpayers. The effectiveness of this policy, of course, leverages the non-Ricardian environment.

Our interest in the interplay of fiscal policy, interest rates, and inflation is motivated in part by the large swings in public debt as a percentage of national income observed in many advanced economies. In Figure 1, we plot public debt as a fraction of GDP for the United States between 1960 and 2022. As is well known, there were sharp increases during the Reagan and George W. Bush administrations as well as during the Great Recession and the COVID-19 pandemic. Conversely, debt declined during the Clinton administration. Figure 1 also plots inflation (dotted line) and a measure of the 5-year real interest rate (dashed line) that controls for demographic trends.⁵ There is no clear link between inflation, real rates, and debt.

A little more nuance can be found by looking at two episodes. Panel A of Figure 2 is a scatter plot of the annual log change in debt-to-GDP (horizontal axis) and inflation (vertical axis) for the period 1960-1988. Panel B is the same period, but with the real 5-year rate on the vertical axis.

⁵Inflation is the four quarter change in the core PCE index. The 5-year real interest rate is defined as the difference between the 5-year Treasury yield and the average 5-year inflation expectations, residualized with life expectancy and fertility. These inflation expectations are based on the average quarterly inflation forecasts over the next 5 years using current inflation.

We see a clear relationship between debt and the real interest rate, but no strong feedback to inflation. Conversely, Panels C and D present the same variables, but for the period 1989-2000. Here, we see a strong relationship between debt and inflation, and a weak relationship with real rates. One natural narrative is that the Federal Reserve under Paul Volker was battling inflation at the same time as Reagan was expanding deficits, generating a bond market equilibrium that adjusted primarily (or exclusively) via changes in nominal and real interest rates. On the other hand, while H.W. Bush and Clinton were reducing debt, the Fed under Greenspan kept real rates high, leading to an equilibrium adjustment primarily through lower inflation.

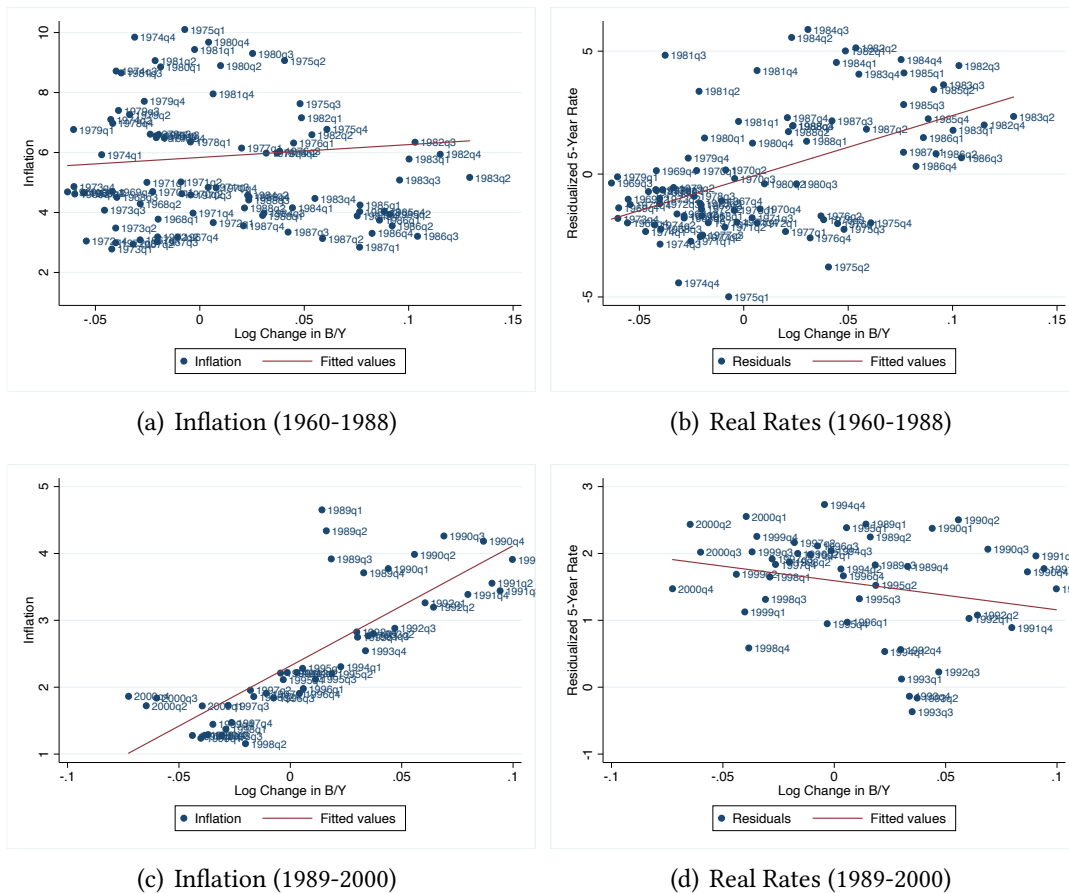


Figure 2: Debt, Real Rates, and Inflation Across Two Episodes

These facts and associated narratives are not presented as a rigorous test of a particular theory. For our purposes, the main takeaway is the non-controversial conclusion that equilibrium adjustments to large changes in government debt stocks work through a combination of prices (inflation and interest rates) and quantities (real income), and the exact mix depends on the policy response. Our contribution is to develop a transparent framework to analyze this interplay and use it to address key policy challenges.

Related Literature

Our work builds on the literature that has integrated the Blanchard-Yaari perpetual youth model into monetary models. Marini and Ploeg (1988) and Cushing (1999) identify monetary non-neutralities in this set-up in the context of flexible prices. Piergallini (2006) and Nistico (2016) study optimal monetary policy in environments with sticky prices and find that strict inflation targeting might no longer be optimal because of the additional financial wealth effects this framework contains. Galí (2021) and Piergallini (2023) focus on the case of low interest rates, $R < G$, and study the implications for asset pricing bubbles and liquidity trap equilibria.⁶ Relative to this work, our contribution focuses on the interactions between monetary and fiscal policy, as in the recent work of Angeletos, Lian, and Wolf (2023). These authors explore how and when fiscal deficits can be “self-financing,” either because they generate a boom in output that raises (proportional) taxes, or (with nominal bonds) because they generate inflation. In recent work, Kaplan, Nikolakoudis, and Violante (2023) explores the mechanics of the fiscal theory of the price level in a heterogeneous agent economy. Our focus is not on how deficits can be self-financing or the fiscal theory of the price level, but rather on the interplay of fiscal and monetary policy in improving the cross-sectional allocation of output via government bonds. Moreover, we study global non-linear solutions which contrast with the first-order approximation approach used in these papers.

Michaillat and Saez (2021) study a New Keynesian framework where the natural rate of interest depends on financial factors. They introduce preferences where households obtain utility from their relative wealth with respect to the population. Our phase diagram is similar to the phase diagram they use to analyze several of the anomalies present in the standard New Keynesian model. Their model however features Ricardian equivalence, and thus it cannot address the effect of changes in the level of government debt, which is our focus.

Our paper is to a large degree motivated by the growing literature on Heterogeneous Agents New Keynesian (HANK) models, which highlights the interaction between fiscal and monetary policies. In HANK models, government debt-financed transfers are non-Ricardian because richer households, who hold a larger share of the debt, have a lower marginal propensity to consume (MPC) than poorer households. Kaplan, Moll, and Violante (2018), for example, shows that the effectiveness of monetary policy depends crucially on the fiscal response. Specifically, monetary expansions are more potent when the government transfers the savings from the interest rate expense back to households. Auclert, Rognlie, and Straub (2018) study fiscal policy multipliers

⁶See also the contributions of Lepetit (2022), Leith and Wren-Lewis (2000), Nistico (2012). We have already mentioned the contributions of Del Negro, Giannoni, and Patterson (2023) and Farhi and Werning (2019) that are also based on a monetary version of the Blanchard-Yaari model.

and highlight the importance of the aggregate intertemporal MPCs, for the magnitude of the multipliers. Other studies, such as McKay, Nakamura, and Steinsson (2016), have examined the impact of forward guidance and the ELB, and found that precautionary savings motives can also temper the power of forward guidance. Our simpler OLG model shares some of the properties of these richer HANK models as in both frameworks households' wealth matters for the determination of interest rates. Our theoretical analysis complements the existing quantitative work and provides insights that can be useful in other applications.

In our framework, government debt matters for the interest rate directly, beyond its effect on aggregate consumption, because the holdings of government debt differ across generations. This is related to findings in Krishnamurthy and Vissing-Jorgensen (2012), that illustrate empirically that government debt carries a convenience yield, which tends to fall with more debt, and that provide a framework to rationalize these findings, where a representative agent values government bonds them in the utility function. Building on this work, Mian, Straub, and Sufi (2022), study how the restrictions from the ELB on monetary policy interact with fiscal policy. Similar to our perpetual youth environment, they find that a low level of government debt can increase the likelihood of a binding ELB equilibrium. However, their emphasis is on the impact of this configuration on the fiscal space of the government rather than on inflation, which never exceeds the target level. Another closely related paper is Bassetto and Sargent (2020), which shares with us the focus on monetary/fiscal interactions. Their example in Section 4.1 showcases the use of the debt Laffer curve, and the result that an optimal policy may stop issuing debt before reaching $r = g + n$ because heterogeneity and distributional concerns. This case fits nicely within our RPI criteria. Finally, a series of papers Caballero, Farhi, and Gourinchas (2017), Caballero and Farhi (2017), and Caballero, Farhi, and Gourinchas (2021) explore the role of government bonds in improving economic outcomes when monetary policy is constrained by the ELB. Our analysis of fiscal policy at the ELB echoes their emphasis on how a shortage of government debt can generate slumps at the ELB and how bonds become an aggregate "demand shifter." We extend the analysis to the impact of debt on global dynamics away from the ELB and to the distribution of resources across heterogeneous agents.

2 Environment

The environment builds closely on the canonical perpetual youth model of Blanchard and Yaari, embedded in the textbook New Keynesian paradigm, as in Galí (2021). Time is continuous and there is no aggregate uncertainty. All announcements will be zero probability "MIT" shocks. A measure of workers supply labor (with a potential lifecycle of earnings), save in a government bond, and are subject to a constant hazard of death, which they insure via annuities. The cru-

cial element for the non-Ricardian aspect of the model is that a new cohort of workers is born every period. On the production side, firms produce differentiated intermediate goods, compete monopolistically, and face quadratic price adjustment costs as in Rotemberg. Workers and the owners of firms (entrepreneurs) are segmented in the sense that workers cannot own shares in firms. This will be useful to separate the return on government bonds from the internal rate of return to private equity. Finally, the government conducts fiscal and monetary policy. In the following subsections, we fill in the details on each block of the model and then characterize the equilibrium.

2.1 Workers

The worker sector closely follows Blanchard (1985) (as well as Buiter, 1988 for the extension to population and technological growth). At any point in time s , a cohort of workers of size $(\lambda + n)e^{ns}$ is born, where n denotes population growth. Each worker faces a constant hazard rate of dying, given by $\lambda > 0$. The expected lifespan of a worker is therefore $1/\lambda$. We require $\lambda + n > 0$.

Letting $\phi(s, t)$ denote the time- t size of the cohort born at $s \leq t$, we have

$$\phi(s, t) = (\lambda + n)e^{ns}e^{-\lambda(t-s)}.$$

The size of the total worker population at time t , denoted m_t , is then

$$m_t = \int_{-\infty}^t \phi(s, t)ds = e^{nt}.$$

A (representative) worker born in period s and alive at $t \geq s$ has preferences given by:

$$\int_t^{\infty} e^{-(\rho+\lambda)(\tau-t)} u(c(s, \tau), l(s, \tau))d\tau, \quad (1)$$

where $\rho > 0$ is the subjective discount factor; $c(s, t)$ is consumption of the final good; and $l(s, t)$ is the amount of labor supplied. Workers effectively discount the future with the sum of the discount factor and the probability of dying. It will be useful to consider the following functional form:

$$u(c, n) = \ln c + \psi \ln(1 - l),$$

with $c \geq 0$ and $l \leq 1$.⁷

⁷Note that we do not restrict $l \geq 0$. A negative l is equivalent to the worker hiring another individual to assist them in “daily living,” removing that unit of time from the production sector, a result that we think is reasonable. Ascari and Rankin (2007) highlighted the possibility of negative labor supply in the perpetual youth framework and proposed using preferences without wealth effects on labor supply to eliminate the possibility.

A worker's productivity z changes over the life cycle. Specifically, define $z_0 \equiv (\lambda + \alpha + n)/(\lambda + n)$. The productivity of a worker born at time s and alive at time $t \geq s$ is given by:⁸

$$z(s, t) = z_0 e^{gt} e^{-\alpha(t-s)},$$

so z declines exponentially with age at rate $\alpha \geq 0$, and grows with time at rate g , where g captures technological growth. Aggregate productivity at time t is then

$$Z(t) \equiv \int_{-\infty}^t \phi(s, t) z(s, t) ds = e^{(g+n)t}.$$

As in Blanchard, workers can perfectly insure their survival risk in spot annuity markets. Let $i(t)$ denote the nominal return on government bonds. For each nominal unit ("dollar") held by a worker in the annuity, they receive $(i(t) + \lambda)dt$ if they survive the next $dt \rightarrow 0$ periods. If they die, the insurance intermediaries receive the asset. As λdt workers die, the insurance sector breaks even with probability one.

Let $P(t)$ be the price of the final good at time t and let $W(t)$ denote the nominal wage per efficiency unit of labor. Workers of cohort s at time t pay non-distortionary taxes $P(t)T(s, t)$. We let

$$T(s, t) = z(s, t)T(t), \tag{2}$$

where T is the tax burden per effective unit of labor. Specifically, we have indexed taxes by cohort in order to allow the tax burden to decline with productivity.⁹ Aggregate tax revenue at t is:

$$\bar{T}(t) \equiv \int_{-\infty}^t \phi(s, t) T(s, t) ds = Z(t)T(t)$$

Let $P(t)a(s, t)$ denote the nominal asset position of the representative agent from cohort s at

⁸ We assume that all cohorts enjoy productivity growth as they age. Instead of modeling growth as a "time effect," we could have assumed that growth is across cohorts:

$$z(s, t) = z_0 e^{gs} e^{-\alpha(t-s)}.$$

As is well known due to the fact that age, cohort, and time have a linear relationship, the two approaches have a simple correspondence. In particular, let $\tilde{\alpha} \equiv \alpha - g$, we have

$$z(s, t) = z_0 e^{gt} e^{-\alpha(t-s)} = z_0 e^{gs} e^{-\tilde{\alpha}(t-s)}.$$

Thus whether technological growth affects all cohorts equally over time or just cohorts at birth (or any linear combination of the two) is covered by the representation in the text.

⁹We do this for simplicity as it facilitates aggregation. Note that the tax (or transfer if negative) remains lump-sum and it is not a function of the labor supply choice.

time t . The flow budget constraint for cohort s is given by:

$$\begin{aligned} \frac{d}{dt} [P(t)a(s, t)] &= \dot{P}(t)a(s, t) + \dot{a}(s, t)P(t) \\ &= (i(t) + \lambda)P(t)a(s, t) + W(t)z(s, t)l(s, t) - P(t)c(s, t) - P(t)T(s, t), \end{aligned}$$

where a “dot” indicates the derivative with respect to time. Dividing through by $P(t)$, we have

$$\dot{a}(s, t) = (r(t) + \lambda)a(s, t) + w(t)z(s, t)l(s, t) - c(s, t) - T(s, t), \quad (3)$$

where $w(t) \equiv W(t)/P(t)$ is the real wage, $r(t) \equiv i(t) - \pi(t)$ is the real interest rate, and $\pi(t) \equiv \dot{P}(t)/P(t)$ is the rate of inflation.¹⁰ Households are subject to the natural borrowing limit, $a(s, t) \geq \underline{a}(s, t)$, which, combined with the log preferences, ensures an interior consumption sequence at an optimum. Letting

$$R(t, \tau) \equiv e^{-\int_t^\tau (r(m) + \lambda) dm}$$

we can integrate the flow budget constraint forward to obtain:

$$a(s, t) = \int_t^\infty R(t, \tau) [c(s, \tau) + T(s, \tau) - w(\tau)z(s, \tau)l(s, \tau)] d\tau. \quad (4)$$

Workers are born with zero wealth; that is, $a(s, s) = 0$ for all cohorts s .

Given a sequence of aggregate taxes $T(t)$ and prices $\{w(t), r(t)\}$, a worker born at time s chooses sequences $\{c(s, t), l(s, t)\}_{t \geq s}$ to maximize (1) subject to (4), with $a(s, s) = 0$, as well as the constraints $c(s, t) \geq 0$, $l(s, t) \leq 1$, and $a(s, t) \geq \underline{a}(s, t)$ for all $t \geq s$, where the natural borrowing limit is

$$\underline{a}(s, t) \equiv - \int_t^\infty R(t, \tau) z(t, \tau) (w(t) - T(t)) d\tau, \quad (5)$$

which is the present value of maximal labor earnings (i.e., $l = 1$) net of taxes. We will require that, in an equilibrium, $\underline{a}(s, t)$ be finite. Note also that $\underline{a}(t, t)$ must be negative, as newborns have no wealth, $a(t, t) = 0$.

The solution to the workers’ problem is characterized as follows:

Lemma 1. *Suppose that $\underline{a}(s, t)$ is finite for all (s, t) . The following conditions characterize the optimal consumption and labor plan of a worker born at s evaluated at $t \geq s$:*

¹⁰For some policy experiments, we may want to consider an unanticipated lump-sum “helicopter” drop of assets (government) bonds to various cohorts at a fixed time t_0 . We will be more explicit about this in Section 2.3, but to streamline the exposition we will suppress this from the notation until needed.

(i) *The Euler equation:*

$$\frac{\dot{c}(s, t)}{c(s, t)} = r(t) - \rho; \quad (6)$$

(ii) *The static labor-consumption condition:*

$$\psi c(s, t) = w(t)z(s, t) (1 - l(s, t)). \quad (7)$$

(iii) *and a consumption function:*

$$c(s, t) = \left(\frac{\rho + \lambda}{1 + \psi} \right) (a(s, t) + h(s, t) - \mathcal{T}(s, t)), \quad (8)$$

where

$$h(s, t) \equiv \int_t^\infty R(t, \tau)z(s, \tau)w(\tau)d\tau$$

represents potential “human wealth” and

$$\mathcal{T}(s, t) \equiv \int_t^\infty R(t, \tau)z(s, \tau)I(\tau)d\tau. \quad (9)$$

represents the present value tax burden.

In the next subsection, we discuss aggregation. We flag a few elements of the individual worker’s problem that will be useful. One is that all cohorts that are alive have consumption that grows at the same rate $(r(t) - \rho)$ and the level of consumption is linear in total wealth net of taxes. A second feature is that given wages, labor supply is linear in consumption, with labor income equal to potential income $w(t)z(s, t)$ minus a fraction ψ of consumption. This feature allows us to express consumption as a fraction financial wealth and potential human wealth.

2.1.1 Aggregation

We now characterize the aggregate behavior of the workers, integrating over the various cohorts. Given that the size of a cohort s at time t is $\phi(s, t)$, and letting capital letters indicate aggregate

quantities, we can integrate over s to define the aggregates:

$$\begin{aligned}
C^w(t) &\equiv \int_{-\infty}^t \phi(s, t) c(s, t) ds \\
N(t) &\equiv \int_{-\infty}^t \phi(s, t) z(s, t) l(s, t) ds \\
A(t) &\equiv \int_{-\infty}^t \phi(s, t) a(s, t) ds \\
H(t) &\equiv \int_{-\infty}^t \phi(s, t) h(s, t) ds \\
\mathcal{T}(t) &\equiv \int_{-\infty}^t \phi(s, t) \mathcal{T}(s, t) ds.
\end{aligned}$$

The following characterizes the aggregate behavior of workers:

Lemma 2. *Given a path $\{w(t), r(t), \bar{T}(t)\}$, worker optimization implies:*

(i) *An aggregate Euler equation:*

$$\dot{C}^w(t) = (r(t) - \rho + \alpha + n)C^w(t) - \frac{(\rho + \lambda)(\alpha + \lambda + n)}{1 + \psi} A(t); \quad (10)$$

(ii) *An aggregate labor supply:*

$$\psi C^w(t) = w(t)Z(t) - w(t)N(t); \quad (11)$$

(iii) *An aggregate consumption function:*

$$C^w(t) = \left(\frac{\rho + \lambda}{1 + \psi} \right) [A(t) + H(t) - \mathcal{T}(t)]; \quad (12)$$

(iv) *And an aggregate evolution of financial wealth:*

$$\dot{A}(t) = r(t)A(t) + w(t)Z(t) - \bar{T}(t) - (1 + \psi)C^w(t). \quad (13)$$

The aggregate labor supply and consumption function, conditions (11) and (12), follow immediately from integrating across cohorts their static decisions. The aggregate dynamics of consumption and financial wealth, however, also have to consider that fraction λ of the population dies every period and a new cohort is born. Those dying in aggregate have average financial wealth $A(t)$ while those being born have zero. The difference in wealth between those dying

and those being born matters for the aggregate Euler equation (10) and the evolution of financial wealth (13). Financial wealth $A(t)$ shows up in (10) because richer agents are replaced by poorer agents; the growth of aggregate consumption is lower with high aggregate wealth. The aggregate Euler equation illustrates that the level of household financial wealth A_t matters for aggregate consumption dynamics, in addition to the interest rate, discount rate, and the age profile of productivity. From (13), aggregate worker wealth evolves “as if” all cohorts inelastically supply $Z(t)$ efficiency units of labor while at the same time spending an extra ψ on consumption. This reflects that for each individual, any increase in consumption reduces labor income at the linear rate ψ via the income effect on labor supply. Note also that Lemma 2 holds for an arbitrary distribution of individual financial assets among surviving cohorts.

Given an initial $A(0)$ and a path for $\{w(t), r(t), \bar{T}(t)\}_{t=0}^{\infty}$, equations (11), (12), and (13) completely characterize the aggregates of the household sector, $\{A(t), C^w(t), N(t)\}_{t=0}^{\infty}$.

2.2 Entrepreneurs

The technology side of the model is familiar from the standard textbook New Keynesian model.¹¹ There is a measure-one continuum of entrepreneurs, each of whom operates a firm that produces a unique intermediate input $j \in [0, 1]$. Intermediate firm technology is given by $y_j(t) = \ell_j(t)$, where ℓ_j are efficiency units of labor. Firms hire these units in a competitive labor market at real wage $w(t)$.

Entrepreneurs sell their output to a competitive final goods sector that combines inputs using a constant-elasticity-of-substitution technology:

$$Y(t) = \left(\int_0^1 y_j(t)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}.$$

Given the constant-returns-to-scale technology and competitive behavior, there is no value added generated by this sector and hence no need to detail the ownership of final-good firms. The price index of the final good is given by

$$P(t) = \left(\int_0^1 p_j(t)^{1-\eta} dj \right)^{\frac{1}{1-\eta}},$$

where $p_j(t)$ is the price of intermediate $j \in [0, 1]$.

For simplicity, and not crucial for what follows, we assume that there are no demographic dynamics for the entrepreneurs: entrepreneurs live forever. They have linear utility and discount

¹¹In particular, this part of the model follows Kaplan, Moll, and Violante (2018) closely. Our Lemma 3 below reproduces their Lemma 1.

at the rate $\tilde{\rho}$. Entrepreneurial wealth consists of shares of its firm. Shares in firms are (potentially) traded among entrepreneurs, but as mentioned above are not available to workers. Moreover, entrepreneurs do not hold government bonds. This is stated as an assumption but is consistent with any equilibrium in which $r(t) < \tilde{\rho}$ for all t .

The entrepreneur's consumption/savings problem is given by:

$$\max_{\{c(\tau)\}_{\tau=t}^{\infty}} \int_t^{\infty} e^{-\tilde{\rho}(\tau-t)} c(\tau) d\tau \quad \text{s.t.} \quad \int_t^{\infty} e^{-\tilde{\rho}(\tau-t)} c(\tau) d\tau \leq V(t),$$

where V is the value of equity held by the entrepreneur at time t . We already impose in the representative entrepreneur's problem that the internal rate of return to equity is $\tilde{\rho}$, which follows from the linearity of preferences and the requirement that consumption be interior in equilibrium. In a symmetric equilibrium, entrepreneurs do not actively trade shares among themselves, and the value of an individual entrepreneur's shares will be equal to the value of their firm, denoted Q .

Entrepreneurs compete monopolistically and choose a sequence of prices to maximize the value of their firm. We restrict attention to symmetric equilibria in which all firms pursue an identical policy.

Intermediate good firms face a nominal friction when setting prices. Let $p(t)$ be the nominal price of an individual variety, where we drop the j index. Intermediate good firms choose the rate of change in their nominal price, $x(t) \equiv \dot{p}(t)/p(t)$ and pay a cost $f(x)Y$, where:

$$f(x) = \begin{cases} \varphi \underline{\pi} x - \frac{\varphi}{2} \underline{\pi}^2 & \text{if } x < \underline{\pi} \\ \frac{\varphi}{2} x^2 & \text{if } x \in [\underline{\pi}, \bar{\pi}] \\ \varphi \bar{\pi} x - \frac{\varphi}{2} \bar{\pi}^2 & \text{if } x > \bar{\pi}. \end{cases}$$

The costs of price adjustment are weakly convex, continuous, and continuously differentiable. For intermediate inflation rates $x \in [\underline{\pi}, \bar{\pi}]$, for some $\underline{\pi} < 0 < \bar{\pi}$, adjustment costs are quadratic, as in Rotemberg (1982). For extreme rates of change, costs are linear. In the spirit of Nakamura and Steinsson (2010), this captures that for high inflation environments, price setting is different than at moderate inflation rates. We shall see that this induces a vertical Phillips curve at extreme inflation.

The entrepreneur chooses a path of p via control $x = \dot{p}/p$ to maximize the value of the firm:

$$Q(t) = \sup_{\{x(\tau)\}_{\tau \geq t}} \int_t^{\infty} e^{-\tilde{\rho}(\tau-t)} [\Pi(p(\tau), \tau) - f(x)Y(\tau)] dt$$

subject to: $\dot{p}(t) = x(t)p(t)$,

where $\Pi(p, t)$ are the real flow profits gross of adjustment costs of a firm charging price p at time t :

$$\Pi(p, t) = \left(\frac{p}{P(t)} - w(t) \right) \left(\frac{p}{P(t)} \right)^{-\eta} Y(t).$$

Note that we assume the government does not tax (or subsidize) entrepreneurs. This rules out implicit transfers to workers through the taxation of entrepreneurs to pay interest on the debt held by workers.¹²

The solution to the optimal pricing plan generates the following Phillips curve:

Lemma 3. *Let $g_Y(t) \equiv \dot{Y}(t)/Y(t)$ denote the real growth rate; $w^* = (\eta - 1)/\eta$ denote the flexible price optimal inverse markup; and $\tilde{\kappa} \equiv \eta/\varphi$. In a symmetric equilibrium with a path for aggregate real wages $\{w(t)\}$, aggregate inflation $\pi(t) \equiv \dot{P}(t)/P(t)$ satisfies:*

$$\dot{\pi}(t) = (\tilde{\rho} - g_Y(t))\pi(t) + \tilde{\kappa} [w^* - w(t)] \quad \text{if } \pi(t) \in [\underline{\pi}, \bar{\pi}]; \quad (14)$$

and for $\pi(t) \notin [\underline{\pi}, \bar{\pi}]$, we have:

$$\begin{aligned} (\tilde{\rho} - g_Y(t)) \underline{\pi} &= \tilde{\kappa} (w(t) - w^*) & \text{if } \pi(t) < \underline{\pi} \\ (\tilde{\rho} - g_Y(t)) \bar{\pi} &= \tilde{\kappa} (w(t) - w^*) & \text{if } \pi(t) > \bar{\pi}. \end{aligned}$$

For interior inflation, $\pi(t) \in [\underline{\pi}, \bar{\pi}]$, the last term in (14) represents the deviation from the flex-price markup, with a positive value indicating that the markup is higher than the flex-price markup. At the extreme rates of inflation, the real wage is uniquely pinned down for any π , generating a “vertical” Phillips curve in $\pi \times w$ space. Let \underline{w} and \bar{w} denote the associated low and high real wages, respectively.¹³

One loose end is that the firm always has the option to shut down production. Flow profits are negative if $f(\pi(t)) > 1 - w(t)$. We rule out equilibria that violate this condition. In particular, this rules out equilibria in which inflation explodes in either direction.

Let $C^e(t) \equiv \int_0^1 c_j(t) dj$ denote the consumption of the entrepreneurial sector. Given paths $\{w(t), \tilde{\rho}\}_{t=0}^\infty$, we say $\{C^e(t), Y(t), V(t), Q(t)\}_{t=0}^\infty$ and $\{\pi(t)\}_{t=0}^\infty$ that solve the entrepreneurs’ problem and satisfy Lemma 3 characterize the entrepreneurs’ sector.

¹²This assumption is made for simplicity, and allows for a simple equilibrium value for aggregate output, as we discuss below.

¹³Specifically, $\underline{w} \equiv \tilde{\rho}\underline{\pi}/\tilde{\kappa} - w^*$ and $\bar{w} \equiv \tilde{\rho}\bar{\pi}/\tilde{\kappa} - w^*$.

2.3 Government

The government sets fiscal and monetary policies under full commitment. Fiscal policy consists of a sequence of non-distortionary aggregate taxes, $\bar{T}(t)$, and real debt $B(t)$, subject to the budget constraint

$$\dot{B}(t) = r(t)B(t) - \bar{T}(t). \quad (15)$$

We assume the government borrows in real bonds promising a real return to differentiate our analysis from the fiscal theory of the price level.¹⁴ We assume a vanishing small amount of nominal bonds that carry the nominal rate $i(t)$ in order to ensure the Fisher equation $i = r + \pi$ holds in equilibrium.

Our policy experiments involve a discrete change to the stock of government debt. For example, suppose at time t_0 the government expands government debt by a discrete amount. The fiscal authority rebates the proceeds to workers. Note that this involves a sale of bonds offset by an aggregate transfer of equal amount. We could circumvent this sale-transfer by assuming the government simply “helicopter drops” the new bonds to households. Specifically, let $\xi(s, t)$ denote the cumulative bond transfers to cohort s as of time $t \geq s$. Let $d\xi(s, t) = \xi(s, t) - \lim_{\tau \uparrow t} \xi(s, \tau)$ denote the amount of new bonds transferred to workers of cohort s at time t . Our experiments assume $\xi(s, t) = 0$ for $t < t_0$, and is constant thereafter. Note that ξ can be negative as well as positive, with negative numbers representing an expropriation of assets. Let

$$d\xi(t) \equiv \int_{-\infty}^t d\xi(s, t) ds$$

denote the aggregate change to worker assets.

We then augment (3) to incorporate the helicopter drop to write:

$$da(s, t) = [(r(t) + \lambda)a(s, t) + w(t)z(s, t)l(s, t) - c(s, t) - T(s, t)] dt + d\xi(s, t).$$

We augment (15) similarly,

$$dB(t) = \left[r(t)B(t) - \bar{T}(t) \right] dt + d\xi(t).$$

As noted already, the evolution of aggregate worker wealth evolves independently of the idiosyn-

¹⁴Although in our current model in continuous time with instantaneous bonds and a price level process that cannot jump (given the adjustment costs), the fiscal theory of the price level will remain inoperative even if the government were to issue nominal bonds.

cratic distribution of wealth, given a path of prices and taxes (see equation 13). Thus, we focus on the aggregate change in government debt, $d\xi(t_0)$, rather than the specifics of the distribution.

Monetary policy is set via an "augmented" interest rate rule:

$$i(t) = \max \left\{ \bar{i} + \theta_\pi \pi(t) + \theta_b \left[\frac{B(t)}{Z(t)} - \frac{B(0)}{Z(0)} \right], 0 \right\}, \quad (16)$$

where 0 is the ELB on nominal interest rates. When $\theta_b = 0$, monetary policy responds only to inflation, as in a standard Taylor rule, and we impose (unless otherwise noted) that the "Taylor principle" $\theta_\pi > 1$ holds. In this case, the intercept \bar{i} is a constant that in the baseline equilibrium corresponds to the level of the real interest rate that is realized when inflation is zero. In our analysis, we also consider a monetary policy rule that can change in response to fiscal policy, hence θ_b . As we will see, this term is useful as fiscal policy affects the real rate in the flexible price allocation.¹⁵

2.4 Definition of Equilibrium

Given a path for fiscal policy $\{B(t), \bar{T}(t)\}$ and monetary policy rule (16), an equilibrium is a path of prices $\{\pi(t) = \dot{P}(t)/P(t)\}$ starting from $P(0) = 1$; real interest rates $\{r(t)\}$ for government bonds; nominal interest rates $\{i(t)\}$; real wages $\{w(t)\}$; firm values $\{Q(t)\}$; and quantities $\{Y(t), C^w(t), C^e(t), N(t), A(t)\}$ such that:

- (i) $C^w(t), N(t)$, and $A(t)$ solve workers' problem given w, r , and T and where $\underline{a}(s, t)$ in (5) is finite for all (s, t) ;
- (ii) $C^e(t), Y(t), Q(t)$, and $\pi(t)$ solve the entrepreneurs' problem given w ;
- (iii) the bond market clears $A(t) = B(t)$;
- (iv) the resource condition is satisfied $C^w(t) + C^e(t) = (1 - f(\pi)) Y(t) = (1 - f(\pi)) N(t)$;
- (v) arbitrage between nominal and real returns by households implies that the Fisher equation is satisfied, $i(t) = r(t) + \pi(t)$; and
- (vi) the monetary policy rule and the government budget constraint are satisfied.

¹⁵There are several papers that study adjustments to the Taylor rule in cases where Ricardian equivalence fails. See for example Curdia and Woodford (2010). Closer to our case, Nisticò (2012) studies a Blanchard-Yaari environment where the "Wicksellian" interest rate deviates (due to preference/demand shocks) from the one of the representative agent.

3 Characterizing Equilibrium Dynamics

In this section, we introduce a phase diagram that will be the main tool to analyze equilibrium dynamics. As a first step, we note a possibly surprising aspect of the equilibrium; namely, effective aggregate labor supply, $N(t)$, and output, $Y(t)$, both grow at an exogenous constant rate $g + n$. To see this, let us use the aggregate evolution of financial wealth (13), together with the budget constraint of the government, and the equilibrium condition $A(t) = B(t)$, and we obtain:

$$w(t)Z(t) = (1 + \psi)C^w(t).$$

The static labor-consumption condition (11) states that $\psi C^w(t) = w(t)Z(t) - w(t)N(t)$. Hence:

$$N(t) = \frac{Z(t)}{1 + \psi},$$

and $Y(t) = N(t)$. In equilibrium, the aggregate labor supply curve is “vertical”; that is, independent of other equilibrium outcomes. This result stems from the balanced growth preferences of workers plus the segmentation of bond markets and taxation. In this model, therefore, fiscal and monetary policies can change the share of output consumed, due to their effect on adjustment costs and inflation, and the share of consumption going to workers versus entrepreneurs, but not the total amount of output produced.

From the workers’ side, recall the aggregate Euler equation (10). Substituting the asset market-clearing condition $A(t) = B(t)$, we obtain

$$\dot{C}^w(t) = (r(t) - \rho + \alpha + n)C^w(t) - \frac{(\rho + \lambda)(\alpha + \lambda + n)}{1 + \psi}B(t).$$

As $B(t)$ increases, in order to clear the bond market we need some combination of a higher $r(t)$, higher $C^w(t)$, or lower growth rate of $C^w(t)$.

As usual, it is convenient to rewrite aggregates in terms of total effective units of labor. In particular, let

$$c(t) \equiv \frac{C^w(t)}{Z(t)}, \text{ and } b(t) \equiv \frac{B(t)}{Z(t)}.$$

As $Z(t)$ grows at rate $g + n$, we have:

$$\dot{c}(t) = (r(t) - \rho - g + \alpha) c(t) - \mu b(t), \tag{17}$$

$$\dot{b}(t) = (r(t) - g - n) b(t) - T(t). \tag{18}$$

where

$$\mu \equiv \frac{(\rho + \lambda)(\alpha + \lambda + n)}{1 + \psi}$$

Finally, $(1 + \psi)c(t) = w(t)$, and letting $c^* \equiv w^*/(1 + \psi)$, we can rewrite the Phillips curve as:

$$\dot{\pi}(t) = \hat{\rho}\pi(t) + \kappa [c^* - c(t)], \quad (19)$$

for $\pi(t) \in [\underline{\pi}, \bar{\pi}]$, and where $\hat{\rho} \equiv \tilde{\rho} - (g + n)$ and $\kappa \equiv (1 + \psi)\tilde{\kappa}$. We postpone the discussion of dynamics for $\pi(t) \notin [\underline{\pi}, \bar{\pi}]$ to below. We will assume that $\tilde{\rho} > (g + n)$, a restriction that guarantees that the firm's pricing problem is well defined.

3.1 Balanced Growth Paths

It will be useful to describe a few properties of a balanced growth path (BGP). In a BGP, all quantities grow at the rate of output, and the real interest rate is constant.

Setting $\dot{c} = 0$, and letting \mathbf{b} , \mathbf{c} , and r denote the corresponding BGP values, we have:

$$\frac{\mathbf{b}}{\mathbf{c}} = \frac{(r - \rho - g + \alpha)}{\mu}. \quad (20)$$

Note that the right-hand side is linearly increasing in r . The interest rate in financial autarky, which occurs when $\mathbf{b} = 0$, is given by $\rho + g - \alpha$, and for higher levels of financial assets we have $r > \rho + g - \alpha$. In a BGP, as debt relative to worker consumption increases, the real interest rate must also increase for the bond market to clear.

Setting $\dot{\mathbf{b}} = 0$, we get from the government sequential budget constraint:

$$T = (r - g - n)\mathbf{b}.$$

In a BGP, $w > T$ for the borrowing limit, $\underline{a}(t, t)$ to be negative. That is, $(r - g - n)\mathbf{b} = T < (1 + \psi)\mathbf{c}$, which requires

$$\frac{(r - g - n)(r - \rho - g + \alpha)}{(1 + \psi)\mu} < 1 \Rightarrow r < \rho + g + n + \lambda,$$

where we used that $r > \rho + g - \alpha$. For $\underline{a}(s, t)$ to be finite for all (s, t) , we also require that $r > g - \alpha - \lambda$, but this is satisfied given $\rho > 0$ and $\lambda \geq 0$ and $r \geq \rho + g - \alpha$. Thus, in a BGP, the interest rate must satisfy:

$$\rho + g - \alpha \leq r < \rho + g + n + \lambda \quad (21)$$

Flexible price interest rates. It will be useful to define the BGP real interest rate associated with the flexible price markup, given debt, $r_*(\mathbf{b})$ as well as its inverse with the following:

$$r_*(\mathbf{b}) \equiv \rho + g - \alpha + \mu \frac{\mathbf{b}}{\mathbf{c}^*} \quad (22)$$

$$\mathbf{b}_*(r) \equiv (r - \rho - g + \alpha) \frac{\mathbf{c}^*}{\mu}. \quad (23)$$

The one remaining element is the determination of the real interest rate $r(t)$, which will depend on the monetary policy rule. We first characterize the case in which monetary policy is not bound by the ELB, and then discuss dynamics in the binding-ELB region of the state space in Section 6.

3.2 Dynamics away from the ELB

Let us consider a fiscal policy that maintains a constant $\mathbf{b}(t) = \mathbf{b}^o$ for all $t \geq 0$. When the ELB is not binding, the Taylor rule (16) becomes

$$i(t) = \bar{i} + \theta_\pi \pi(t).$$

Here, we assume the target inflation rate is zero, which requires $\bar{i} = r_*(\mathbf{b}^o)$. For the ELB not to bind we require $i(t) > 0$, which implies $\pi(t) > -\bar{i}/\theta_\pi$. This will be the range of inflation relevant for this subsection.

The Taylor rule, combined with the Fisher equation implies $r(t) = i(t) - \pi(t) = \bar{i} + (\theta_\pi - 1)\pi(t)$. Substituting into (17), we have

$$\dot{\mathbf{c}}(t) = (\bar{i} + (\theta_\pi - 1)\pi(t) - \rho - g + \alpha) \mathbf{c}(t) - \mu \mathbf{b}^o. \quad (24)$$

Equations (19) and (24) are two ordinary differential equations (ODEs) in $\pi(t)$ and $\mathbf{c}(t)$. This system of two equations, combined with the condition that inflation is bounded and $\mathbf{c} \geq 0$ in equilibrium, characterize all possible equilibria in which the ELB does not bind.

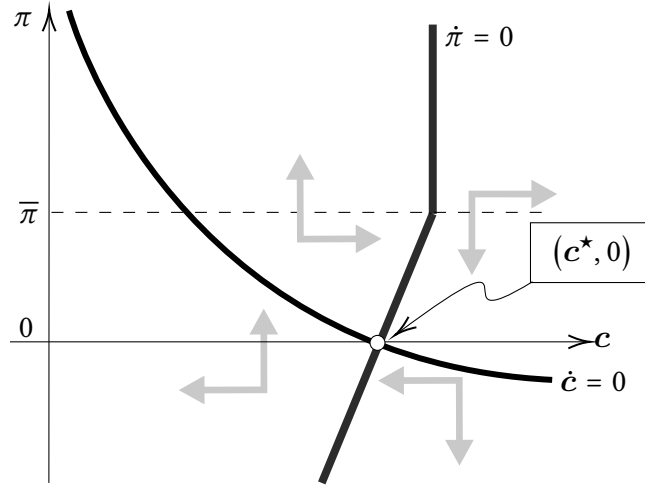
To analyze the dynamic system, we use the phase diagram in Figure 3. The curve labelled “ $\dot{\pi} = 0$ ” sets $\dot{\pi}$ in (19) to zero:

$$\pi = \frac{\kappa}{\hat{\rho}} (\mathbf{c} - \mathbf{c}^*) \quad \text{for } \pi \in [\underline{\pi}, \bar{\pi}]. \quad (25)$$

As \mathbf{c} increases above \mathbf{c}^* , firms would like to raise their markup, which is counterbalanced by the costs of adjusting prices faster. The stationary point trades off higher π against higher \mathbf{c} .

Along the $\dot{\pi} = 0$ curve inflation is constant, at a value that increases with \mathbf{c} . Above this locus, $\dot{\pi} > 0$, and below we have $\dot{\pi} < 0$. These dynamics are represented by the arrows pointing up and

Figure 3: Phase Diagram: Dynamics Away From the ELB



down in the phase diagram.

Outside of $[\underline{\pi}, \bar{\pi}]$, the lower terms in (14) imply that the Phillips curve is vertical. There is a subtlety when it comes to dynamics for $\pi \notin [\underline{\pi}, \bar{\pi}]$. Along the vertical portion of the $\dot{\pi} = 0$ locus, constant inflation is consistent with profit maximization, but so are movements along the vertical section, as firms are indifferent about the choice of π .

The curve labelled “ $\dot{c} = 0$ ” is the locus of points at which $\dot{c} = 0$. From (24), we have:

$$\pi = - \left(\frac{\bar{i} - \rho - g + \alpha}{\theta_{\pi} - 1} \right) + \left(\frac{\mu}{\theta_{\pi} - 1} \right) \frac{b^o}{c}. \quad (26)$$

As c increases, the bond market requires a lower real interest rate to clear. Given that $\theta_{\pi} > 1$, this implies a lower rate of inflation and an even lower nominal interest rate. This generates a negative relationship between c and π in order to keep $\dot{c} = 0$.

From (24), we see that as π increases for a given c relative to the $\dot{c} = 0$ locus, $\dot{c} > 0$. These dynamics are depicted by the horizontal arrows in Figure 3. Note that when $b^o = 0$, the $\dot{c} = 0$ curve would be horizontal with inflation equal to the first term in (26).

Again, care must be taken for $\pi \notin [\underline{\pi}, \bar{\pi}]$. For inflation outside this interval, there are unique levels of consumption, \underline{c} and \bar{c} , that are consistent with firm optimization. Thus, there are no consumption dynamics in equilibrium outside $[\underline{\pi}, \bar{\pi}]$; that is, if $\pi \notin [\underline{\pi}, \bar{\pi}]$ is part of an equilibrium trajectory, c is constant while the economy moves along the relevant vertical portion of the $\dot{\pi} = 0$ curve.

The intersection of the two curves is the zero-inflation steady state, denoted by $(c^*, 0)$, which is the target of the monetary policy. This requires the Taylor rule intercept \bar{i} to be set to $\bar{i} =$

$r_*(b^o)$, so that the bond market clears at the zero-inflation steady state.¹⁶ Note that achieving the zero inflation outcome requires an intercept that depends on the level of debt, which reflects the non-Ricardian environment, and anticipates our discussions of fiscal and monetary policy coordination. The zero inflation steady state is unstable. In particular, the eigenvalues of the linearized system evaluated at the steady state both have real parts strictly greater than zero.

It is possible that in our model the ELB binds. In the next section, we will however abstract from this possibility and postpone the discussion of the ELB and the implications for fiscal and monetary policy interactions until Section 6.

4 Bond Demand and Robust Pareto Improvements

This section considers whether and how increasing the stock of government debt affects the distribution of consumption and welfare in our New Keynesian environment with heterogeneous agents.

We begin by noticing that the only aggregate store of value for workers in this economy is government bonds. So let us first discuss the role of government debt in their consumption demand. To think through this, consider a BGP where $T > 0$. In this case, currently alive generations who own government bonds effectively own a claim to future government tax revenues. Differently from the Ricardian equivalence case, those tax revenues are coming (partially) from generations that are not yet alive and will be born with no bonds. Once they are born, they will be taxed and the proceeds given to bond holders, generating a redistribution of resources *across generations*. As a result, the consumption (per units of effective labor) of older generations than own bonds is higher than those who are just born, and the difference depends on the level of debt. This clarifies why government bonds represent net wealth for bond holders, and enter the aggregate Euler equation (10). This also shows why the timing of taxation matters in this model, as Blanchard (1985) explained.

More related to the inefficiency we will study below is that this difference of consumption across generations continues to arise even when the government is no longer taxing, but rather *transferring resources* (when $r < g + n$ so that $T < 0$). In this case, an owner of a bond perceives that she/he will be able to sell the bond at a high price in the future to generations not yet alive. This future bond demand from generations not yet born represents the “social contrivance” of Samuelson (1958) and encapsulates the main inefficiency of the OLG environment, something we will discuss in detail below.

¹⁶Note that $\bar{i} = r_*(b^o) > \rho + g - \alpha$ for $b^o > 0$, and thus the $\dot{c} = 0$ lines crosses the horizontal axes. In addition, the plot as drawn requires that the ELB does not bind at the intersection, that is, $\bar{i} = r_*(b^o) > 0$. We will discuss the case where the ELB binds at the zero-inflation steady state later in Section 6.

In Aguiar, Amador, and Arellano (2022) we showed that a strong demand for government bonds so that r is less than the growth rate can also occur in an Aiyagari (1994) model and that it opens scope for a “robust” Pareto improvement (RPI) via a government bond issuance. We defined an RPI to be a policy that induces a change in prices and taxes such that the budget set of any agent is guaranteed to be weakly expanded at any state and time. One advantage of the RPI criteria is that it ensures a Pareto improvement regardless of how agents tradeoff consumption intertemporally or across uncertain states. In an RPI, with weakly greater flow income at all dates, any agent’s initial equilibrium consumption path remains affordable. Thus their welfare cannot fall and must increase if the budget set expands strictly. A second advantage of the RPI criteria is that, to check for the existence of an RPI, only knowledge about how aggregate private savings respond to real interest rate changes is needed. The potentially rich household heterogeneity only affects the conditions for the existence of an RPI through the shape and interest rate elasticity of this aggregate savings response.

In the present model, that elasticity is endogenous and partially controlled by the monetary policy response to fiscal policy. This raises the question of whether monetary policy introduces an additional tool to achieve an RPI with respect to a real model. In this section, we show that the answer is no.

Though our model has aggregation, the RPI provides a useful starting point to discuss the existence of Pareto improvements in models with more heterogeneity where aggregation does not obtain. Since the primary mechanism for a Pareto improvement is the private sector demand for government bonds, these lessons may extend to the now popular quantitative heterogeneous agent New Keynesian (HANK) models. For completeness, later we also discuss policies leading to Pareto improvements without necessarily being an RPI.

4.1 RPIs in the Real Model

Let us start by considering the version of our model without price rigidities. In this case, some immediate results obtain: in all equilibria, and for all $t \geq 0$, $c(t) = c^*$, $w(t) = (1 + \psi)c^*$, and output and total effective labor supply remain as before, $Y(t) = N(t) = Z(t)/(1 + \psi)$.

Suppose now that we start from an initial BGP with $\mathbf{b}(0) = \mathbf{b}^0 \geq 0$ and a corresponding real rate $r^0 = r_*(\mathbf{b}^0)$. We will consider a simple policy change: at $t = 0$, the government issues new debt, moving the total stock of debt (in efficiency units) to \mathbf{b}' from \mathbf{b}^0 , and adjusts taxes on workers to satisfy its budget constraint.¹⁷ After $t = 0$, there are no additional policy changes, debt remains constant at \mathbf{b}' , and taxes are adjusted to satisfy the government budget constraint. From equation (17), it follows that $r(t)$ must also be constant, at a value we denote by r' . From equation (18) it

¹⁷As noted above, the case of a bond issuance is equivalent to the government distributing bonds directly to the existing worker cohorts, increasing aggregate savings by $\mathbf{b}' - \mathbf{b}^0$.

follows then that transfers remain constant at a value T' . Taken together, this means that after the policy change, the economy jumps to a new BGP, and there are no transitional dynamics.¹⁸

The question we ask is whether no worker or entrepreneur has a reduction in income at any point in time, and at least one agent has a strict increase, as a result of this policy change. If so, this is an RPI, and hence a Pareto improvement.

Note that at the moment of the policy change, the government, given its budget constraint, must transfer $\mathbf{b}' - \mathbf{b}^o$ of resources (or tax, if negative) to the cohorts of workers alive at $t = 0$. An RPI requires that all workers have a weakly higher flow income at every date, including $t = 0$, and thus, $\mathbf{b}' \geq \mathbf{b}^o$. The flow income of a worker of cohort s in subsequent periods, after the policy change, is :

$$(r' + \lambda)a + w^*z(s, t)l - z(s, t)T'$$

where a and l represent the asset and labor supply choices for cohort s at t ; and r' , and T' represent the new interest rate and tax levels; and where we have used that the wage rate remains constant. In this real version of our model, the flow income of entrepreneurs is unchanged and equals $\Pi(t) = (1 - w^*)Y(t)$.

We define an RPI as follows:

Definition 1 (RPI in the real model). *We say that the fiscal policy of increasing debt from \mathbf{b}^o to $\mathbf{b}' > \mathbf{b}^o$ generates an RPI in the real model if (i) $T' \leq T^o$ and, (ii) for any (s, t) , $r'a^o(s, t) \geq r^oa^o(s, t)$, where $a^o(s, t)$ is the original equilibrium choice of assets for cohort s at time t .*

If the new equilibrium satisfies these conditions, given that the initial transfer to workers is strictly positive ($\mathbf{b}' > \mathbf{b}^o$), then the original consumption and labor supply decisions remain feasible for workers after the policy change; their welfare, therefore, cannot fall and has strictly increase for the generations alive at $t = 0$. Moreover, the welfare of the entrepreneurs is unchanged. These conditions are then sufficient for a Pareto improvement. An important element here is that it is not necessary to detail how the aggregate consumption \mathbf{c}^* is reallocated across workers.¹⁹

The fact that the original consumption path is feasible for all workers is sufficient to guarantee that the original allocation is Pareto dominated. In the proof of Lemma 4, we show that in the BGP, $a(s, t) \geq 0$ for all s and t . That is, no cohort has a negative asset position in the initial

¹⁸The lack of transitional dynamics is in our view a main difference with respect to Bewley-Aiyagari type models, where the distribution of wealth needs time to converge to a new ergodic state after a policy change. In our model, the distribution of wealth does not need to adjust after the policy change, a feature driven by the linearity of the policy functions. A potential extension of the present model, that will generate transitory dynamics, is to introduce additional heterogeneity on the workers' side as in Gertler (1999).

¹⁹This was our main motivation when introducing the RPI in the context of a Bewley-Aiyagari model. It is not necessary to specify how agents trade off consumption across states and time. As will be true here as well, only the aggregate savings supply is needed to determine the feasibility of an RPI.

equilibrium, and thus condition (ii) requires $r' \geq r^o$.²⁰

We can combine the government budget constraint at the new level of debt, b' , and the aggregate Euler equation, using the definition of b_\star in (23), to obtain the following relation between taxes, debt, and the real rate

$$T' = (r' - (g + n))b_\star(r'). \quad (27)$$

where $b_\star(r') = b'$. Note that $b_\star(r')$ is the aggregate savings supply schedule of the economy in the new BGP, and uniquely determines r' , given b' .

Now, a local increase in b from b^o must be matched by an increase in r from r^o . For this to generate a *decrease* in taxes, T , it suffices that

$$-b'_\star(r^o) \frac{r^o - (g + n)}{b^o} > 1 \quad (28)$$

Thus, if *the aggregate savings elasticity with respect to the real interest rate* (net of $n + g$) is higher than unity, then an RPI exist. Given that $b'_\star(r^o) > 0$, it follows from (28) that $r^o < g + n$. This is not sufficient however, $b'_\star(r^o)$ must also be large enough.

The elasticity condition (28) is related to the debt Laffer curve: it guarantees that the economy starts from a region below the peak of the curve, and thus it is possible to increase government revenue by increasing debt.²¹ Figure 4 shows the debt Laffer curve and its relationship with the existence of an RPI. In our current model, we can go a bit farther, as we can characterize the peak of the Laffer curve in closed form:

Lemma 4 (Existence of an RPI in the Real Model). *Suppose the real model is in an initial BGP. There exists a debt issuance at $t = 0$ that generates an RPI if and only if $r^o - g - n < (\rho - \alpha - n)/2 < 0$.*

In an RPI, while each agent has (weakly) more flow income, there is no change in aggregate output. The increase in the interest rate needed to clear the bond market ensures that aggregate worker consumption does not change. The welfare improvement comes from a better distribution across cohorts of the same amount of aggregate income.

²⁰Every cohort has a consumption profile that changes at a constant rate, $r^o - \rho$, and a labor earnings profile (net of taxes) that changes at a constant but weakly lower rate, $g - \alpha \leq r^o - \rho$. The latter is a requirement for the economy to sustain non-negative levels of government debt in the stationary equilibrium, see equation (20). Given that newborns have zero assets, it follows that the asset holdings of every cohort must remain non-negative over their lifecycle.

²¹As mentioned in the introduction, several other papers have studied the debt Laffer curve in the context of $r < g$. See for example Bassetto and Sargent (2020) and Mian, Straub, and Sufi (2022). The first is very related. In particular, their discussion in their Section 4.1 represents an example of an RPI within an OLG model, although focused on steady states.

Figure 4: The Debt Laffer Curve and RPIs in the Real Model

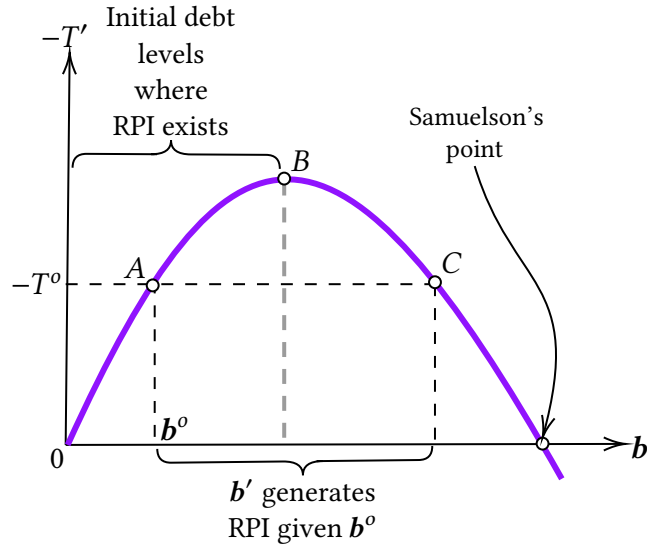


Figure 5: The horizontal axis represents the new levels of debt, b' , and the vertical axis represents the negative of the corresponding tax levels, T' . For any initial value of b^0 that lies from the origin to point B, there exists an RPI. Given the b^0 plotted, the ranges of b' from point A to C represent a corresponding RPI. Samuelson's point is explained in Subsection 4.3.

We highlight here that an RPI arises from a simple policy change: an increase in debt, and a corresponding reduction in taxes. For example, we do not require sophisticated changes in the tax system: the spirit of the RPI approach is that very little information about how agents value trade-offs is required to engineer a Pareto improvement. Also, the condition for the existence of an RPI only requires knowledge of the aggregate savings response to interest rates. In the simple environment we are analyzing here, we could have directly exploited our knowledge of the agents' preferences to check for a Pareto improvement (something that we will do later on). But in more complex environments, with richer agent heterogeneity, such welfare comparisons may become harder.

As we discussed above, what is critical for the existence of an RPI is whether the real interest rate increases sharply (or not) with government debt issuances. A sharp increase in real rates leads to a reduction in resources after the increase in debt and may necessitate increasing taxes (or reducing transfers) to satisfy the government budget constraint, violating the RPI conditions. One can imagine that monetary policy can help in this calculation: the monetary authority has the capacity to keep real rates low after a fiscal expansion (at least for a while), and thus it seems this can facilitate the region of existence of RPIs. That is, in the monetary model, the elasticity of savings to real interest rates is no longer an exogenous parameter, but it is determined in equilibrium and affected by the monetary policy rule. In addition, such monetary intervention

may also have downsides, as inflation may change. We proceed to study this next.

4.2 RPIs in the Monetary Model

We now return the model with nominal rigidities. We assume that we start in a BGP, which we indicate by superscript “o” with zero inflation, so that $\pi^o = 0$ and thus $c^o = c^*$ for some $b^o \geq 0$. This requires that $r^o = \bar{i} = r_*(b^o)$.

We again consider a simple policy change: at $t = 0$, the government changes the total stock of debt to b' from $b^o \geq 0$ and adjusts taxes on workers to satisfy its budget constraint. We assume that the ELB is not binding and characterize the new BGP that arises as the result of the policy.²² The monetary policy rule is given by

$$i' = \bar{i} + \theta_\pi \pi' + \theta_b (b' - b^o).$$

The parameter θ_b controls the response of monetary policy to government debt. The other equilibrium conditions necessary to determine the new BGP are the Fisher equation, the aggregate Euler, and the Phillips curve:

$$i' = \pi' + r', \quad b' = \frac{r' - \rho - g + \alpha}{\mu} c', \quad \pi' = \frac{\kappa}{\hat{\rho}} (c' - c^*),$$

where we are assuming that $\pi' \in (\underline{\pi}, \bar{\pi})$, so we remain away from the vertical part of the Phillips curve. Using the above, we can solve for the level of taxes at the new BGP as a function of the aggregate savings schedule as follows

$$T' = (r' - (g + n)) b_m(r') \tag{29}$$

where $b_m(r')$, the aggregate savings supply schedule in the monetary economy, is now given by:²³

$$b_m(r') = b_*(r') \left[1 + \frac{(\theta_b^* - \theta_b)(r' - r^o)}{\theta_b(r' - r^o) + \theta_b^*(\gamma + \theta_b b^o)} \right]$$

²²For the ELB not to bind at the initial BGP, it must be that $r_*(b^o) \geq 0$. Given that we are assuming that the ELB is not binding after the policy change, the economy must transfer to a new BGP, given the unstable dynamics.

²³We can also recover the levels of π' and c' in the new BGP. Those are:

$$c' = c^* \left[1 + \frac{(\theta_b^* - \theta_b)(r' - r^o)}{\theta_b(r' - r^o) + \theta_b^*(\gamma + \theta_b b^o)} \right] \text{ and } \pi' = \frac{\kappa (\theta_b^* - \theta_b)(r' - r^o) / \hat{\rho}}{\theta_b(r' - r^o) + \theta_b^*(\gamma + \theta_b b^o)}.$$

with $\theta_b^* \equiv \frac{\mu}{c^*}$ and $\gamma \equiv \frac{(\theta_\pi - 1)\kappa}{\hat{\rho}c^*}$.

A comparison of T' in (29) with T' in the real model, (27), reveals an important distinction: the shape of the aggregate savings supply schedule is now a function of the monetary policy rule coefficients, as well as the Phillips curve parameters.

Just as in the real model, at the moment of the policy change, the government must transfer an additional $b' - b^o$ of resources (or tax, if negative) to workers. To make sure that we have an RPI, we require then that $b' > b^o$. The flow income of a worker of cohort s in subsequent periods, after the policy change, is :

$$(r' + \lambda)a + w'z(s, t)n - z(s, t)T'$$

where a and n represent the asset and labor supply choices for cohort s at t ; and r' , w' and T' represent the new prices and tax levels. Differently from the real model, the markup now may respond to the policy change. The flow income of an entrepreneur is profits minus price adjustment costs:

$$\Pi' - f(\pi') = (1 - w' - f(\pi')) Y' = \hat{\rho}Q',$$

where π' and Y' represent the new inflation and output levels. We can now define an RPI:

Definition 2 (RPI in the monetary model). *We say that the fiscal policy of increasing debt from b^o to $b' > b^o$ generates an RPI in the monetary model if (i) $T' \leq T^o$, (ii) for any (s, t) , $r'a^o(s, t) \geq r^oa^o(s, t)$, where $a^o(s, t)$ is the original equilibrium choice of assets for cohort s at time t , (iii) $w' \geq w^o$, and (iv) $\Pi' - f(\pi') \geq \Pi^o - f(\pi^o)$.*

The first two conditions are the same as in the real model. The last two conditions are new. Workers' savings are non-negative, $a(s, t) \geq 0$ for all s and t , by the same argument as in the real model. Thus condition (ii) requires $r' \geq r^o$. The third condition requires that the wage rate does not fall, and the fourth condition requires that the profits do not fall. All of them together guarantee that the fiscal expansion generates a Pareto improvement. But given that output is constant, the last two conditions are potentially contradictory.

When $\theta_b = \theta_b^*$, the savings supply schedule of the monetary model and the real model coincide. From the above equations, we have that $c' = c^o = c^*$, $w' = w^o = w^*$, and $\pi' = 0$; and thus, with this monetary policy rule, the new BGP coincides with the new BGP in the real model. The last two conditions of Definition 2 are automatically satisfied, and it follows:

Lemma 5 (RPI in the Monetary Model). *Suppose the monetary model with $\pi^o = 0$ and with $\theta_b = \theta_b^*$. Then there exists a debt issuance at $t = 0$ that generates an RPI under the same conditions as in Lemma 4.*

Recall that we are focusing on a case where the ELB is not binding in the initial BGP: that

is, $r^o > 0$ given $\pi^o = 0$. The real rate is always weakly higher than the autarkic interest rate: $r^o \geq (\rho + g - \alpha)$, as $b^o \geq 0$. For the condition in the Lemma 4 to hold, it must be that $\rho + g - \alpha < r^o < g + n + (\rho - \alpha - n)/2$ with $r^o > 0$. The condition for the existence of an RPI together with $r^o > 0$ requires then:

$$n > \rho - \alpha > -(n + 2g), \quad (30)$$

which will hold for sufficiently large n and g . This condition is also sufficient in the following sense: if condition (30) holds then there is a BGP with $b^o > 0$ such that an RPI exists.²⁴

For any value of $\theta_b \in [0, \theta_b^*)$, we have that $b_m(r') > b_\star(r')$ and $b'_m(r^o) > b'_\star(r^o)$. In this case, the monetary model features a total savings schedule that is more elastic to interest rates than the real model. Thus, the range of debt levels for which debt issuances reduce taxes has expanded in comparison. But to achieve this, the monetary policy necessary boosts inflation after the fiscal expansion, and as a result, the share of output that is allocated to labor increases: $w' > w^o$ and $\pi' > 0$. Both of these *reduce* entrepreneurial profits. Thus, *the fiscal policy does not generate an RPI unless the monetary policy sets $\theta_b = \theta_b^*$.*

In Figure 6, we illustrate two distinct debt Laffer curves for the monetary economy for two distinct values of θ_b . The solid line corresponds to $\theta_b = \theta_b^*$, and it is the same Laffer curve as in the real model. A monetary policy rule with a $\theta_b = 0$, increases the elasticity of the aggregate savings schedule at b^o to the interest rate, and thus, the peak of the Laffer curve shifts to the right.

To summarize, in our model, monetary policy does not create a free lunch since it inevitably redistributes resources between workers and entrepreneurs. We did not let the government to compensate entrepreneurs through subsidies for potential profit losses due to a fiscal expansion. And a more accommodative monetary policy rule could generate additional fiscal revenue to fund such subsidies and thereby facilitate the existence of an RPI. Allowing transfers between entrepreneurs and workers makes the model less tractable, and for this reason, we leave this question for future work.

4.3 Pareto Improvements

The RPI result relies on a relaxation of all the agents (all cohorts of workers and all entrepreneurs) flow budget constraints, guaranteeing that their welfare is weakly higher after the policy change. An alternative approach for evaluating the feasibility of Pareto improving policies is to directly evaluate agents' welfare, and such an approach can also provide more intuition on the nature of

²⁴This follows from the continuity of $r_\star(b)$. In the absence of population growth and technological growth, condition (30) cannot be satisfied. This is because of $\pi^o = 0$. Starting from a BGP with $\pi^o > 0$ will again open the possibility of an RPI with a non-binding ELB.

Figure 6: The Debt Laffer Curve and RPIs in the Monetary Model

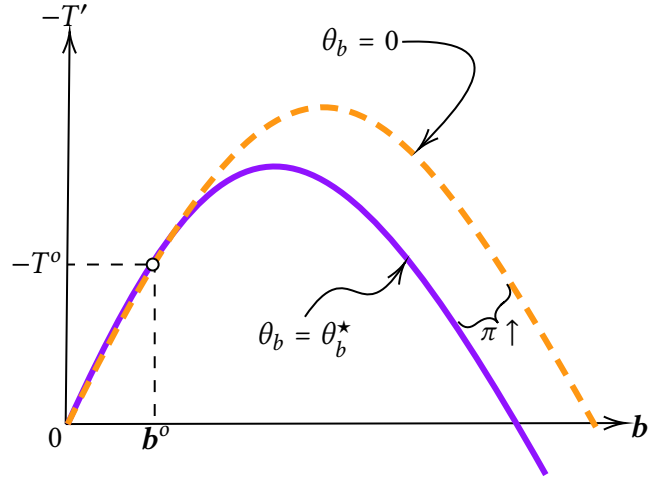


Figure 7: The horizontal axis represents the new levels of debt, b' , and the vertical axis represents the negative of the corresponding tax levels, T' . The curves represent the levels of transfers in the new BGP for two different monetary policy rules. The solid curve is the case where $\theta_b = \theta_b^*$ which replicates the real economy. The dashed curve is the case with $\theta_b = 0$, where the monetary policy rule facilitates larger transfers, but at the cost of higher inflation and lower profits.

the improvement. This will take us back to the classical analysis of Samuelson (1958).

First, let us consider how a worker's welfare varies with the real interest rate (and the wage) in a BGP. In particular, consider a newborn worker born at time t in a BGP:

Lemma 6. *In a BGP with real interest rate r and real wage w , a newborn worker at time t has utility*

$$U^t(r, w) = \frac{1 + \psi}{\rho + \lambda} \left[\log(\rho + \lambda + g + n - r) + \frac{r - \rho}{\rho + \lambda} + \frac{\log w}{1 + \psi} + \frac{gt}{1 + \psi} + u_0 \right],$$

where u_0 is a combination of parameters.

One can see that the newborn's welfare at time t is increasing in the wage, which represents the share of output going to workers. Their welfare also depends non-monotonically on r . A greater r implies lower discounted lifetime wealth, lowering period t consumption, but a faster growth rate of consumption going forward. For $r \in [\rho - \alpha + g, \rho + \lambda + g + n)$, the range of rates consistent with a BGP according to (21), U^t is a strictly concave function in r with a maximum at $r = g + n$:

Corollary 1. *For a given wage, the welfare of a newborn worker in a BGP is maximized when $r = g + n$.*

To see the intuition for this result, consider the following (stationary) planning problem. Let $\chi(\Delta)$ denote the share of aggregate worker consumption that is allocated to the generation born Δ periods ago. At time t , this means that cohort s consumes $c(s, t)$ which must satisfy $\phi(s, t)c(s, t) = \chi(t - s)C^w(t)$ where $C^w(t) = wZ(t)/(1 + \psi)$. In addition, it must be that

$$\int_t^\infty \chi(t - s)dt = 1 \Leftrightarrow \int_t^\infty \frac{\phi(s, t)}{Z(t)} c(s, t)dt = \frac{w}{1 + \psi}$$

Consider now the problem of the planner maximizing the utility of a newborn at time t subject to the constraint above:

$$\int_t^\infty e^{-(\rho+\lambda)(t-s)} \log c(s, t)dt \text{ s.t. } \int_t^\infty e^{-(\lambda+g+n)(t-s)} c(s, t)dt = \frac{e^{g_s} w}{(1 + \psi)(\lambda + n)}$$

where we have used the values of $\phi(s, t)$ and $Z(t)$. The solution to this problem is the same as the solution to an inter-temporal problem of a newborn individual that faces a real interest rate equal to $g + n$.

In an equilibrium, individual workers may see a net return to bonds different than $g + n$. For example, the autarkic interest rate (when $\mathbf{b} = 0$) is $\rho + g - \alpha$ which may be less than the value of $g + n$ for n large enough. From their Euler equation, an $r \neq g + n$ distorts the inter-temporal path of consumption relative to the social optimum, which in turn distorts the cross-sectional allocation of the exogenous amount of resources. This allows for an increase in the supply of bonds to improve upon the equilibrium allocation by raising the interest rate and facilitating inter-generational trades.

Newborn welfare is maximized at $r = g + n$, but a Pareto improvement must also account for existing cohorts at the time of the policy change. If $\mathbf{b}^0 = 0$, then existing cohorts have zero wealth and are thus identical to newborns in regard to the welfare consequences of a change in r , but in addition get the initial distribution $\mathbf{b}' - \mathbf{b}^0$. However, if $\mathbf{b}^0 > 0$, then existing cohorts have positive wealth and are thus sensitive to changes in r beyond the effects on newborn cohorts. The positive savings give them an extra benefit from higher real interest rates. To see this, let us derive the utility of workers of cohort s at t in a BGP with prices r and w . As derived in the proof of Lemma

6, this is given by

$$U(s, t) = \frac{1 + \psi}{\rho + \lambda} \left[\log \left(\frac{a(s, t)(\alpha + \lambda + n)(\rho + \lambda)}{we^{gt}z_0} + (\rho + \lambda + g + n - r)e^{-\alpha(t-s)} \right) + \frac{r - \rho}{\rho + \lambda} + \frac{\log w}{1 + \psi} + \frac{gt}{1 + \psi} + \frac{\psi}{1 + \psi} \alpha(t - s) + u_0 \right],$$

where u_0 is the same combination of parameters as in the lemma. For a given level of $a(s, t) \geq 0$, utility is strictly increasing in the interest rate for $r < g + n$. Contrary to newborns, if $a > 0$ utility peaks at r strictly greater than $g + n$, due to the presence of positive assets. This implies the feasibility of Pareto improving policies when $r^0 < g + n$. These Pareto improvements, as with the RPI discussion above, also require a combination of fiscal and monetary policies, because as in Lemma 5, their coordination is necessary to guarantee that the utility of entrepreneurs and workers weakly increase. In particular, the monetary policy rule needs to react to the fiscal expansion to maintain the level of inflation unchanged at $\pi^0 = 0$.

In Figure 4 we denote the point where $r = g + n$ as the "Samuelson's point". At this point, the economy is at a Pareto optimum with respect to the allocation of resources among workers. For values of debt below this point, the equilibrium allocation is Pareto dominated by Samuelson's point. However, if the economy started from a lower level of debt and policy moved it towards this point, although interest rates would increase towards $g + n$, transfers do not uniformly increase. As the Figure shows, transfers decrease with debt for levels of debt to the right of the peak of the transfer curve. Such a decline in transfers is the main reason increasing debt up to the Samuelson's point does not constitute an RPI.

When $r^0 < g + n$, workers perceive a return on savings that differs from the true social return. For $r^0 < g + n$, an increase in b increases real rates which benefits all generations of workers. But this is not enough for a Pareto improvement, because depending on the monetary policy rule, an increase in b may generate inflation, affecting entrepreneurs' welfare. The following lemma summarizes this result.

Lemma 7. *Suppose the economy is in an initial BGP with $r^0 < g + n$. Then there exists a new BGP with a combination of fiscal and monetary policies that is a Pareto improvement.*

We want to highlight that the key inefficiency in the model behind Lemmas 4, 5, and 7, is the presence of a never-ending flow of new generations of workers (or the "infinite hotel" of Shell, 1971). This is the seminal insight of Samuelson (1958), and there is a simple way to see it in our current environment. Set $\alpha = 0$, and consider a situation without new cohorts being born. This corresponds to the case where $n = -\lambda$, that is, population shrinks at the death rate λ . In that

case, $A(t)$ drops out from the aggregate Euler equation (10).²⁵ There is then a unique real rate consistent with a BGP, $r = \rho + g > g + n$, and thus, there is no role for government bonds in improving upon (or even affecting) the market equilibrium.

So far we have assumed the initial BGP is away from the ELB. If the initial equilibrium is at the ELB, the Pareto improvement also has the potential to take the economy away from the ELB. In Section 6, we explore in detail these dynamics.

This subsection extends the insight of Samuelson to a New Keynesian environment.²⁶ The crucial lesson learned here is that expanding the stock of safe assets is not sufficient to improve welfare. In the New Keynesian model, this will have potentially unappealing inflationary and distributional consequences. However, the pairing of debt issuance with an appropriate monetary rule that responds to the fiscal expansion rescues the traditional insight that increasing the quantity of safe assets may improve the distribution of a fixed amount of income. As noted above, this insight may have implications for the broader HANK literature.

The previous discussion have assumed that the economy remained away from the vertical part of the Phillips curve. We conclude this section with a brief analysis of their role.

Large Monetary Accommodations and the Non-Linear Phillips Curve

Recent work by Angeletos, Lian, and Wolf (2023) studies situations where a fiscal expansion can be self-financing.²⁷ We now briefly discuss a related situation in the context of our model.

In our previous analysis, we showed that only when $\theta_b = \theta_b^*$ and initial debt is below the peak of the debt Laffer curve is a fiscal expansion consistent with an RPI. For values of θ_b between 0 and θ_b^* , a fiscal expansion still increases the real interest rate, but at a lower pace than when $\theta_b = \theta_b^*$, and inducing a redistribution of resources and inflation.

However, the monetary rule could respond strongly enough to a fiscal expansion so as to lower the real interest rate. This means even if the initial real rate is above $g + n$, requiring positive taxes in the initial BGP, the new BGP after a fiscal expansion could feature low enough real rates to switch fiscal policy from *positive taxes to positive transfers*.

This situation can occur if these two conditions are satisfied. First, the autarkic interest rate

²⁵Buiter (1988) referred to this as the debt neutrality case. Note that our equations do not work in the limit of $n = -\lambda$ when $\alpha > 0$, as there is no effective aggregate amount of labor left in the long-run in this case.

²⁶Balasko and Shell (1980) extended Samuelson (1958) to provide precise conditions on the existence of Pareto improvements in OLG models. In the stationary context, Balasko and Shell show that an equilibrium is Pareto efficient if and only if $r \geq g + n$. This subsection extends that result to the standard New Keynesian framework, including the necessary adjustment of the monetary policy rule. Moreover, these improvements can be made with simple policies that do not rely on complex changes to the tax schedule.

²⁷Their analysis contains additional elements not included in ours (such as the possibility of inflating away part of the initial nominal debt and increasing fiscal revenue given a constant tax rate). But it is interesting to ask a similar question to theirs in our model.

must lie below $g + n$, that is $\rho - \alpha < n$. Second, θ_b must be sufficiently negative:

$$\theta_b < -\frac{\kappa\mu(\theta_\pi - 1)}{\rho(n - \rho + \alpha)}.$$

An expansion of debt under these conditions would not constitute an RPI, but provides an extreme example of a fiscal expansion that makes the entire fiscal debt self-financing.

Consider now the case of $\mathbf{b}' \rightarrow \infty$ in the new steady state, which implies under these conditions that the real interest rate converges to a value less than $g + n$, if we ignore the non-linear part of the Phillips curve. Underlying this limit, the corresponding $\mathbf{c}' \rightarrow \infty$; thus it ceases to be a valid equilibrium for sufficiently high \mathbf{b}' , the economy must be operating at the vertical region of the Phillips curve. We can obtain a sufficient condition so that, independent of the monetary rule, the real rate in any BGP with $\mathbf{b}' > \mathbf{b}^o$ cannot be lower than $g + n$. Thus no monetary policy rule can turn a BGP tax into a BGP subsidy after a fiscal expansion. For this it suffices that:

$$r^o > g + n + (n - \rho + \alpha) \left(\frac{\bar{c}}{\mathbf{c}^*} - 1 \right),$$

where $\bar{c} > \mathbf{c}^*$ is the maximal consumption of workers consistent with the Phillips curve.

5 Monetary and Fiscal Policy Interactions

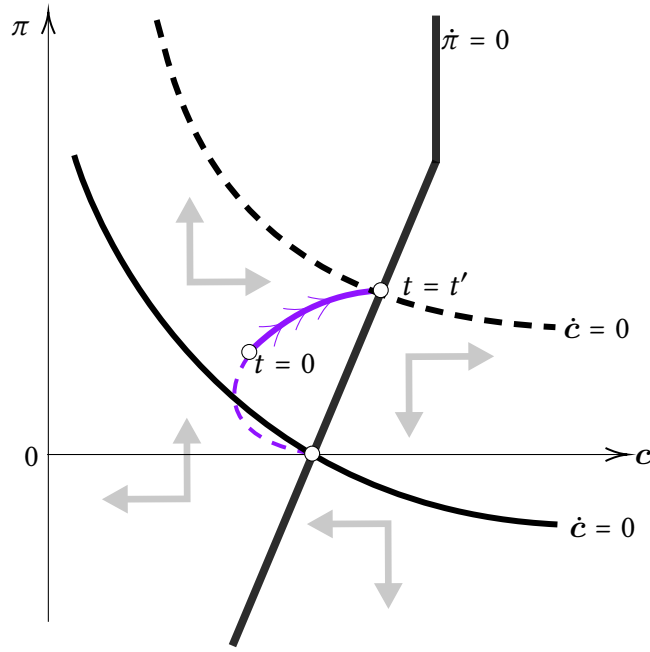
In this section, we use the phase diagram to explore equilibrium dynamics for various scenarios and revisit some results in the literature. Our two scenarios involve (i) an anticipated fiscal deficit/government bond issuance; and (ii) the forward guidance puzzle. In the next section, we explore an additional scenario on the use of fiscal policy to escape a binding ELB. In all of these scenarios, we set $g = n = 0$, that is, no technological or population growth. We do this for simplicity.

5.1 Anticipated Deficits

Our first analysis involves an anticipated debt-financed tax cut (or transfer increase). Many politicians run on such a platform, including US Presidents Reagan and George W. Bush, as well as, most recently, the mini-budget of UK Prime Minister Elizabeth Truss. We show that anticipated deficits can increase inflation and may increase or decrease consumption on impact, depending on the horizon and the response of monetary policy.

We initialize the current period as $t = 0$ and assume we are at the zero inflation steady state with some level of debt \mathbf{b}^o for $t < 0$. At $t = 0$, there is an unanticipated announcement that at

Figure 8: Response to an Anticipated Bond Issue



time $t' > 0$ the government will increase debt to $b' > b^0$ and rebate the proceeds to workers. We trace out the path of inflation and worker consumption until the economy reaches the new steady state. In doing so, we focus on the dynamics away from the ELB.

In Figure 8, we replicate the phase diagram from Figure 3 and we first study the case when $\theta_b = 0$. At t' , the “ $\dot{c} = 0$ ” shifts out due to the change in b , which is the dashed line labeled $\dot{c} = 0$. The size of this shift for a given b' depends on parameters, in particular λ , ρ , and α . Note that in the Ricardian case of $\alpha = \lambda = 0$, there is no shift. Prior to t' , the economy is still subject to the dynamics governed by the original b^0 , which are depicted by solid lines and the arrows. The Phillips curve does not depend on b , and hence the “ $\dot{\pi} = 0$ ” remains stable.

The economy jumps to the trajectory at $t = 0$ and travels along that path until it reaches the new steady state at exactly $t = t'$. For larger t' , the announcement effect places the economy closer to the original steady state; for smaller t' , the economy jumps closer to the eventual steady state. Depending on parameters, the eigenvalues may be complex or real, and the resultant path may cycle or not, respectively.

We depict a thick solid portion of the trajectory as an example path. The initial point involves higher inflation and lower worker consumption. This is combined with positive \dot{c} and $\dot{\pi}$. The economy's response can be understood through the logic of the bond market (which is the flip side of the goods market). A higher eventual worker consumption (and hence $\dot{c} > 0$) lowers the demand for the initial (fixed) stock of bonds for standard inter-temporal substitution reasons. The bond market clears at $t = 0$ with a higher real interest rate, which raises the demand for bonds.

The higher real interest rate must be accompanied by higher inflation, due to the monetary policy rule: $r = \bar{r} + (\theta_\pi - 1)\pi$, with $\theta_\pi > 1$. Whether the jump in π is associated with an increase or decrease in the $t = 0$ worker consumption depends on the time horizon and the parameters of the model.

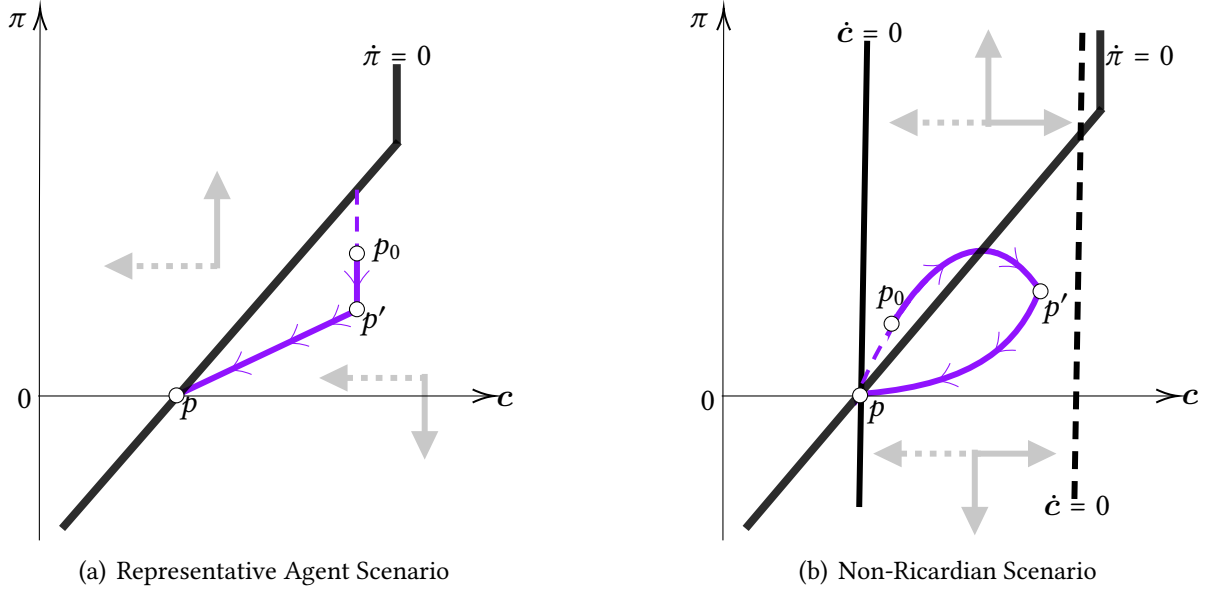
For the experiment of Figure 8, we considered the case of a monetary rule with $\theta_b = 0$, which holds the intercept constant at $r_\star(\mathbf{b}^0)$. The increased \mathbf{b} requires an increase in the real interest rate, but given the interest rate rule, this must be associated with higher inflation. If the monetary authority increased its intercept at t' , the shift in the $\dot{c} = 0$ curve would be dampened. In particular, the central bank can keep the economy at the zero inflation initial steady state for all $t > 0$ by promising to increase the intercept (and hence the nominal interest rate) one-for-one with the necessary increase in the real interest rate to absorb the new bonds at the initial level of income. In particular, this outcome is possible by setting $\theta_b = \theta_b^\star$ in the rule. The economy is at the zero inflation steady state for all $t > 0$, and worker consumption is unchanged. The only effect of the fiscal expansion is to increase the level of debt and a rotation of the $\dot{c} = 0$ at time $t = t'$.

The primary conclusion from this analysis is that in a non-Ricardian environment, the economic consequences of anticipated deficits hinge on the response of the monetary authority's rule to the increase in debt. If the central bank follows the policy advice obtained from the Ricardian benchmark that the long-run target real interest rate is invariant to fiscal policy, the consequences are higher inflation and avoidable fluctuations in consumption. To an outsider well versed in the Ricardian literature, the central bank appears to be doing exactly as prescribed; namely, following a set monetary rule that leans against lax fiscal policy. However, the rigidity of the rule is cause rather than cure for the inflation observed in equilibrium.

5.2 The Forward Guidance Puzzle

The phase diagram is also a transparent analytical tool to understand the “forward guidance puzzle” of Del Negro, Giannoni, and Patterson (2023) and McKay, Nakamura, and Steinsson (2016). In a standard representative agent New Keynesian model, Del Negro, Giannoni, and Patterson (2023) showed that an announcement to temporarily reduce real interest rates at some point in the future had a large effect on consumption and inflation in the announcement period, and all periods leading up to the interest rate cut. Moreover, the further in the future the reduction would happen, the larger the initial effect. Using alternative quantitative models, Del Negro, Giannoni, and Patterson (2023), McKay, Nakamura, and Steinsson (2016), and Kaplan, Moll, and Violante (2018) showed that breaking Ricardian equivalence mitigates this puzzle. Del Negro, Giannoni, and Patterson (2023) builds on Blanchard-Yaari, and hence is closest to our framework. The other

Figure 9: Forward Guidance Puzzle



two papers build on Aiyagari (1994), but draw similar lessons.

We can adapt the McKay, Nakamura, and Steinsson (2016) experiment to our setting as follows. Let r^o be the stationary real interest rate $r_\star(\mathbf{b}^o)$ that clears the bond market at zero inflation (equation 22). At $t = 0$, the monetary authority announces that at $t_0 > 0$ it will reduce the *real* interest rate below r^o for the interval $[t_0, t_1)$, before resuming its original policy. This is equivalent to interest rate rule with $\theta_\pi = 1$, $\theta_b = 0$, and an intercept that shifts down for $t \in [t_0, t_1)$. Specifically, for some $\Delta > 0$, monetary policy follows:

$$i(t) = \begin{cases} r^o + \pi(t) & \text{for } t \notin [t_0, t_1) \\ r^o - \Delta + \pi(t) & \text{for } t \in [t_0, t_1). \end{cases} \quad (31)$$

This policy means that for $t < t_1$, the monetary authority is implementing a particular path of the real interest rate. After t_1 , we assume that the economy is back at the zero inflation steady state.

To replicate the representative agent “puzzle,” consider the case with zero government bonds. From (10), we recover an aggregate Euler Equation that is identical to that of a representative agent economy. The stationary real interest rate is $\rho - \alpha$, which for the current experiment will be r^o .

Panel (a) of Figure 9 depicts the phase diagram for the standard forward guidance puzzle. With zero government bonds, (17) becomes

$$\dot{c} = (r(t) - \rho + \alpha)c(t). \quad (32)$$

In this case, any level of c is a potential steady state, as long as $r(t) = \rho - \alpha$. When $r(t) = r^o - \Delta < \rho - \alpha$, we have $\dot{c} < 0$. The $\dot{\pi} = 0$ curve remains unchanged from the previous scenarios.

The equilibrium can be solved backwards from t_1 . At $t = t_1$, the real interest rate returns to $r^o = \rho - \alpha$, and the economy must be back at its zero inflation steady state, labeled p . For $t \in [t_0, t_1)$, we can solve (32) with $r(t) = r^o - \Delta$ backwards in time from the boundary condition $c(t_1) = c^*$. Associated with this path $c(t)$ there is a path for $\pi(t)$ that satisfies (14) with the boundary condition $\pi(t_1) = 0$. This trajectory is depicted as the path leading from the point labeled p' to the zero-inflation steady state p .

For $t \in [0, t_0)$, the monetary authority sets $r(t) = r^o = \rho - \alpha$, and $\dot{c} = 0$. The economy at $t = 0$ thus jumps to a point directly above p' and follows a vertical trajectory that reaches p' at $t = t_0$. This is depicted as point p_0 . Note that at announcement, the economy jumps to a point with higher inflation and higher c . Moreover, the further in the future is t_0 , keeping $t_1 - t_0$ constant, the longer time is spent on the vertical trajectory. As $t_0 \rightarrow \infty$, the economy starts closer and closer to the $\dot{\pi} = 0$ line and spends longer in the high inflation-high consumption situation. The puzzle is illustrated by the implication that the further in the future the reduction is, the larger the initial effect on inflation and the longer the economy is with high inflation and high consumption.

To see how breaking Ricardian equivalence mitigates this sensitivity to forward guidance, consider the case with $b^o > 0$. The monetary policy continues to be given by (31), but now $r^o = r_*(b^o)$, the stationary real interest rate given by (22).

The phase diagram is depicted in Panel (b) of Figure 9. Given the monetary rule, the $\dot{c} = 0$ line is vertical at c^* for $t \notin [t_0, t_1)$. We depict this baseline locus and the associated dynamics for $c(t)$ with a solid line and solid arrows, respectively.

For $t \in [t_0, t_1)$, from (17) the $\dot{c} = 0$ line is given by

$$c = \frac{(\alpha + \lambda)(\rho + \lambda)b^o}{r^o - \Delta - \rho + \alpha}.$$

Here, we assume that $0 < \Delta < r^o - \rho + \alpha$; otherwise there is not a well defined stationary locus for the low-interest environment. This line is depicted by the dashed vertical locus, and the associated trajectories for c are depicted by the dashed arrows. The dynamics for inflation are the same regardless of $r(t)$.

To explore the equilibrium response to forward guidance, we again work backwards through time. The trajectory from p' to p characterizes the path during $t \in [t_0, t_1)$. The dynamics are governed by the dashed arrows, and the economy must reach p at $t = t_1$. In particular, at $t = t_0$, the economy is on the trajectory that leaves from the intersection of the dashed $\dot{c} = 0$ locus and the $\dot{\pi} = 0$ and leads to point p . The precise point along this trajectory is determined by the length of time $t_1 - t_0$.

Prior to t_0 , the economy follows the dynamics governed by the solid arrows. These trace out a path that leaves from p to p' . At the announcement, the economy jumps to a point p_0 on this trajectory that ensures arrival at p' at $t = t_0$. The further in the future is t_0 , keeping $t_1 - t_0$ constant, the longer the span spent on this trajectory, and the closer to the original (and final) steady state p the economy starts at $t = 0$. Hence, there is no longer a “forward guidance puzzle”: The further in the future is the planned interest rate cut, the less the initial economy reacts.

The crucial difference between panels (a) and (b) is that the representative agent economy can spend an indefinite amount of time at *any* level of consumption, as long as $r(t) = \rho - \alpha$. Conversely, in the non-Ricardian scenario, the stationary interest rate depends on b/c , and hence there is a unique $c = c^*$ that is stationary, given b^o . This is a property shared by the HANK models of McKay, Nakamura, and Steinsson (2016) and Kaplan, Moll, and Violante (2018). Moreover, given the unstable dynamics around the zero-inflation steady state, the dynamics pick up speed as we move away from p , and hence long trajectories must start close to that steady state.

5.2.1 Alternative Fiscal Policies and Forward Guidance

In Farhi and Werning (2019), the authors use a Blanchard-Yaari model and argue that incomplete markets (or the presence of new generations in our model) are not sufficient to resolve the forward guidance puzzle. In this section we contrast this result from the result we have obtained above, discuss the distinct modelling assumptions that lead to the difference in outcomes, and show the role that fiscal policy plays in determining the efficacy of forward guidance.

In the model of Farhi and Werning (2019), there is no government debt.²⁸ Instead, there is a Lucas tree that can be traded among the households. This Lucas tree provides a dividend that is proportional to the level of output. We can do something similar in our model, but operating through fiscal policy. In particular, we assume that the government collects a proportion of labor income, δ , *through the lump-sum tax*.²⁹ The parameter δ is constant over time. At the beginning of time, the government distributes an asset that is a claim to this future tax revenue. In subsequent dates, the government collects the tax revenue and distributes it to the owners of the asset as a dividend. This asset plays the role of the Lucas tree, but can also be interpreted as a government bond under a particular issuance policy, a discussion we postpone for the end of this section. Let

²⁸The authors present several models. We will focus on their version of the Blanchard-Yaari model with rational expectations.

²⁹This avoids introducing a distortion in labor supply. But we could also assume that the government sets a proportional tax on labor income, in which case, aggregate effective labor supply would be

$$N(t) = \frac{1 - \delta}{1 + \psi - \delta} Z(t).$$

The rest of the analysis remains the same, as the system in (33) does not change.

us denote by $A(t)$ the value of this asset at time t . Assuming that the value of the asset is the present value of the dividends, we can write

$$\dot{A}(t) = r(t)A(t) - \delta w(t)N(t).$$

This valuation equation now takes the role of the government budget constraint in the analysis.

We can continue as before and derive the aggregate Euler equation, which is the same as in (10). Letting $\mathbf{a}(t) \equiv A(t)/Z(t)$, and $\mathbf{c}(t) \equiv C^w(t)/Z(t)$, we obtain the normalized Euler equation and a normalized valuation equation:

$$\begin{aligned} \dot{\mathbf{c}}(t) &= (r(t) - \rho + \alpha) \mathbf{c}(t) - \frac{(\alpha + \lambda)(\rho + \lambda)}{1 + \psi} \mathbf{a}(t), \\ \dot{\mathbf{a}}(t) &= r(t)\mathbf{a}(t) - \delta \mathbf{c}(t), \end{aligned} \tag{33}$$

and where we have used that $C^w(t) = w(t)N(t)$ (the market clearing conditions for the final good).

For comparison to the paper, we will assume that prices are fully rigid, and thus $\dot{\pi} = 0$ at all times (which is equivalent to assuming that the Phillips curve is completely horizontal in the exercises we will perform). Thus the system of equations (33) is all that is needed to characterize the equilibria.

This system features a trivial BGP at $\mathbf{c} = \mathbf{a} = 0$, which we will ignore. The other non-trivial BGPs can be found by setting $\dot{\mathbf{c}} = \dot{\mathbf{a}} = 0$, and require that the real rate $r(t) = r^*$ where r^* is the unique positive value such that

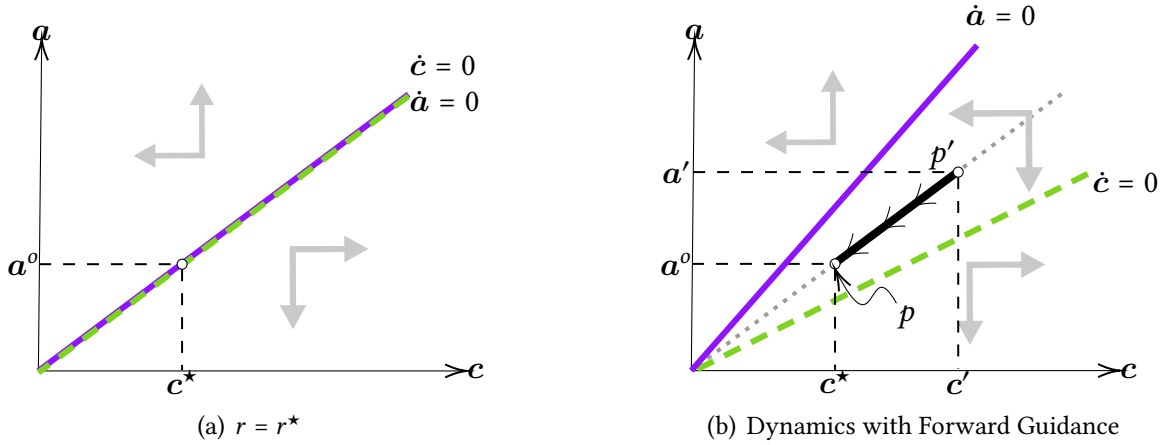
$$\frac{r^* - \rho + \alpha}{(\alpha + \lambda)(\rho + \lambda)} = \frac{\delta(1 + \psi)}{r^*}.$$

When $r(t) = r^*$ there is a linear manifold of pairs (\mathbf{c}, \mathbf{a}) that constitute a BGP (again, ignoring the non-linear part of the Phillips curve). This is not surprising given that the right-hand sides of (33) are linear; but it contrasts with our model, where the BGP (ignoring the ELB) given a constant real rate is unique as long as $\mathbf{b} > 0$. Panel (a) of Figure 10 depicts the phase diagram when $r = r^*$ and pinpoints the BGP where $\mathbf{c} = \mathbf{c}^*$. Given that locus $\dot{\mathbf{c}} = 0$ and the locus $\dot{\mathbf{a}} = 0$ are the same, any point in the line is also a BGP.

We now have the ingredients to describe the effect of forward guidance in this modified model. We assume that the economy started originally from the BGP where $\mathbf{c} = \mathbf{c}^*$ with an $r = r^*$. As before, the policy is announced at the beginning of time: the real interest rate would be reduced at t_0 and maintained at that rate until t_1 . To solve for an equilibrium we assume that the economy returns to the original BGP after the interest rate has recovered at t_1 .³⁰

³⁰This is harder to justify in this case as any point in the manifold can be sustained indefinitely, but keeps us close

Figure 10: Forward Guidance Puzzle: An Alternative Fiscal Policy



The reduction in the real rate has an effect on the BGP locus $\dot{c} = 0$ and $\dot{a} = 0$. In particular, the locus associated with $\dot{c} = 0$ rotates to the right in the (c, b) space as illustrated in Figure 10 panel (b). The locus associated with $\dot{a} = 0$ rotates to the left in the same space. The dynamics that operate in the system from t_0 to t_1 are illustrated by the large arrows in panel (b).

Now we can describe the effect of forward guidance. At time $t = 0$, there announcement of the future reduction of the real rate at time $t = t_0$ moves the economy from point p to point p' (as shown in panel b). The valuation of the asset increases from a^0 to a' , and workers consumption expands from c^* to c' . The economy remains stationary at p' until $t = t_0$. Once the reduction of the real interest rate occurs at time $t = t_0$, the economy starts to move along the center line downwards towards point p , as denoted by the arrows. Point p is reached exactly at $t = t_1$, representing the end of the interest rate reduction. The amount of of time spent between t_0 and t_1 determines the increase in a and c at the announcement. But note that there are no dynamics before $t = t_0$: the horizon of the interest rate changes does not matter for the effect on output of the announcement.³¹

The dynamics shown in panel (b) are similar to the dynamics that would arise in a representative agent model. That is, c jumps on announcement, remains constant as the interest rate has not yet changed, then slowly decreases back to c^* starting at $t = t_0$ (following the Euler equation with a lower real rate). At time $t = t_1$ the economy is back to its starting point. In the representative agent case, the horizon of the change in interest rates does not affect the change in consumption at announcement (as in the analysis in the Figure 9 panel a). So, in the present model, the forward guidance puzzle remains as strong as it was in the corresponding representative agent model.

to the assumption made in the literature.

³¹This is equivalent to the finding in Farhi and Werning (2019) that the interest rate elasticity of output is independent of the horizon of the interest rate change. A reader may note that this is exactly so because point p' lies in the original BGP locus, a result that arises from the linearity of system (33).

We can now reinterpret the valuation equation as a budget constraint. That is, consider instead a situation where a represents the amount of government debt outstanding. And the government raises lump-sum taxes/transfers to manage its budget. The equilibrium behavior in Figure 10 panel (b) can now be read as follows: At announcement, the government issues new bonds, and moves the economy to point p' . For a period of time before $t = t_0$, the government rolls over the debt and just pays interest on its bonds. Once the interest rate change occurs at $t = t_1$, the government then reduces the stock of outstanding debt (using the fact that the interest rate has just decreased). It does so until it reaches the original level of debt at the BGP at $t = t_1$.

An interesting lesson arises: the strength of forward guidance depends on the associated fiscal policy. In particular, an expansionary fiscal policy makes forward guidance more effective. The lesson should not be a surprise as, for example, the effects of monetary policy depend on the underlying assumptions about fiscal policy in HANK models (Kaplan, Moll, and Violante, 2018; Kaplan, Moll, and Violante, 2016).³²

The use of forward guidance as policy tool arose out of the need to enhance the toolkit of central banks when facing the zero lower bound constraint on nominal interest rates. The exercise in this last section highlights that fiscal policy can help make forward guidance more effective. But also hints that fiscal policy on its own may be sufficient to boost the economy out a liquidity trap. We proceed to study this case in the next section.

6 The Effective Lower Bound

In the previous sections, we have abstracted away from the possibility of the zero lower bound on nominal interest being binding. In this section, we complete the model description by stating the dynamics of the economy including the ELB, and discussing the monetary and fiscal policy interactions at the bound. For completeness, we let $g \neq 0$ and $n \neq 0$.

The phase diagram in Figure 3 depicts equilibrium trajectories when the monetary authority is unencumbered by the ELB. As noted above, this will be the case for $\pi(t) > -\bar{i}/\theta_\pi$. For $\pi(t) < -\bar{i}/\theta_\pi$, the monetary rule cannot be followed without running afoul of the ELB.

When the ELB binds, the nominal interest rate is zero, and the Fisher relation implies $r(t) = -\pi(t)$. Substituting this into (17), we have

$$\dot{c}(t) = (-\pi(t) - \rho - g + \alpha) c(t) - \mu b^o. \quad (34)$$

³²Farhi and Werning (2019) discuss how having liquidity (in our model, $A(t)$) that co-moves with income (in our model, workers labor income) is important in their result. We reinterpret this from the point of view of our model by noting that the amount of liquidity is a policy choice, driven by fiscal policy.

Hence, for $\pi(t) < -\bar{i}/\theta_\pi$, the stationary points for c are given by:

$$\pi = -(\rho + g - \alpha) - \mu \frac{b^o}{c}. \quad (35)$$

When the ELB binds, we have a positive relationship between π and c that keeps $c(t)$ constant. A lower real interest rate is obtained by higher inflation under the ELB, and hence bond-market clearing requires a higher c for a higher π when $\dot{c} = 0$. Note that equation (34) does not depend on any aspects of monetary policy but is sensitive to the amount of government debt. We return to this in Section 6 when we discuss how to use fiscal policy to escape the ELB.

In Figure 11 we add this additional locus to that of Figure 3. The horizontal line labeled *ELB* demarcates the threshold $\pi = -\bar{i}/\theta_\pi$. Note that if $b^o = 0$, both schedules are independent of c , which would be the case in a Ricardian environment in which the stationary real interest is independent of the level of debt and income.

From (34), we see that as π increases for a given c relative to the $\dot{c} = 0$ locus, $\dot{c} < 0$. Thus, the direction of change for c and π are the same between the two $\dot{c} = 0$ loci, regardless of whether we are above or below the ELB threshold. Hence, there is no discontinuity in trajectories at the ELB threshold.

Figure 11 depicts a situation where there are two stationary points. The first steady state is that depicted already in Figure 3, at which the ELB does not bind. At the other steady state, the ELB is binding, and the dynamics around it are saddle path stable. The existence of this steady state, and the dynamics around it, follows the logic spelled out in Benhabib, Schmitt-Grohé, and Uribe (2001).³³

6.1 Discount Factor Shocks and Avoiding the ELB

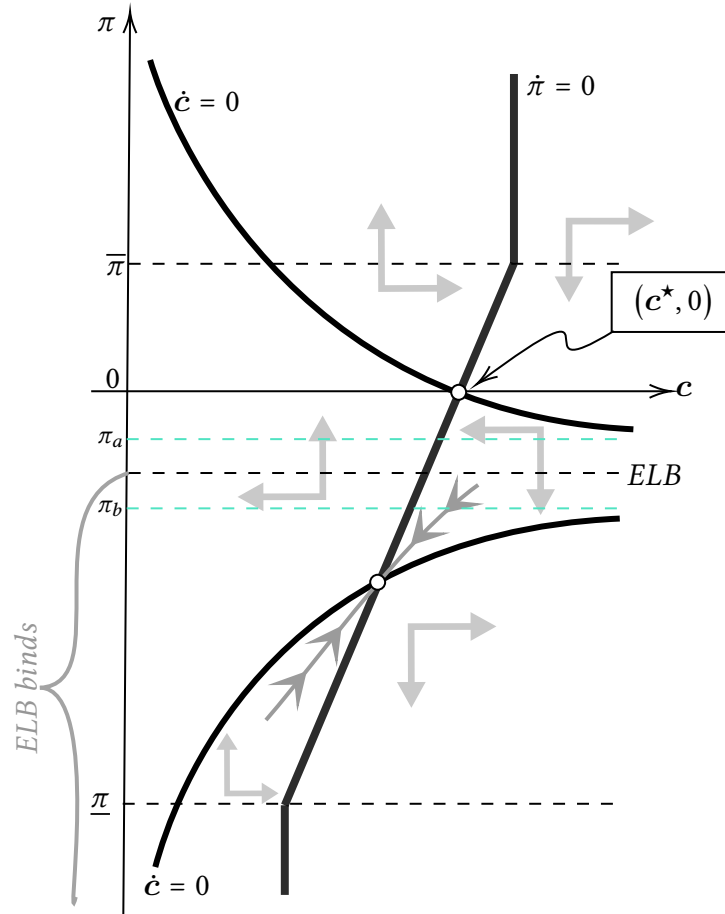
In standard New Keynesian models, a transitory increase in patience can generate a deflationary recession and perhaps a binding ELB.³⁴ These typically stand in for exogenous declines in consumer demand. We now explore such shocks in our framework, map out the dynamics, and show how fiscal policy can avoid the ELB. For simplicity, we will focus on analyzing cases where the intercept of the monetary policy rule, $\iota(t)$, may directly depend on the time-varying parameters and fiscal policy and assume $\theta_b = 0$.

Let workers' discount rate be denoted $\rho(t)$, which takes on two values $0 < \underline{\rho} < \bar{\rho}$ and for

³³ Note that it is possible to have further steady states in the ELB region. If the lower part of the $\dot{c} = 0$ curve had intersected the $\dot{\pi} = 0$ curve twice, we would have a third steady state, which would be unstable, and if three times (which is possible due to the vertical section of the Phillips curve), then there is an additional stable point, as well. The fact that there could be multiple possible steady states in the ELB region is a consequence of the failure of Ricardian equivalence in the model.

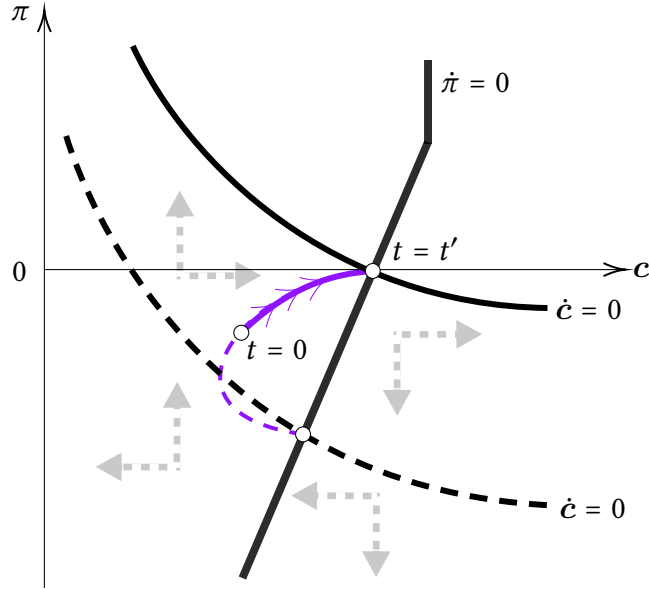
³⁴For example, see Krugman (1998), Eggertsson and Woodford (2003), and Jung, Teranishi, and Watanabe (2005).

Figure 11: Phase Diagram



Note: ELB represents the value $\pi = -\frac{i}{\theta_\pi}$. For inflation levels below this value the ELB constraint is binding; while it is not above. The values of π_a and π_b represent the asymptotes of the respective $\dot{c} = 0$ lines. Their values are $\pi_a = \frac{i - (\rho + g - \alpha)}{\theta_\pi - 1}$ and $\pi_b = -(\rho + g - \alpha)$. The graph is drawn for the case where $\pi_a > -\frac{i}{\theta_\pi} > \pi_b$.

Figure 12: Temporary Decline in ρ : ELB does not bind



some $t' > 0$ follows:

$$\rho(t) = \begin{cases} \bar{\rho} & \text{for } t \notin [0, t') \\ \underline{\rho} & \text{for } t \in [0, t'). \end{cases}$$

That is, for $t < 0$, $\rho(t) = \bar{\rho}$. At $t = 0$, $\rho(t)$ unexpectedly declines to $\underline{\rho} < \bar{\rho}$. For $t \in [0, t')$, $\rho(t) = \underline{\rho}$. For $t \geq t'$, the discount rate returns to $\rho(t) = \bar{\rho}$. The change at $t = 0$ is unexpected, but the equilibrium path is one of perfect foresight thereafter.

We consider two scenarios, both of which assume that we are at the zero-inflation steady state for $t < 0$. The first case studies a mild increase in patience, such that the zero-inflation steady state remains viable in equilibrium, conditional on an appropriate response of monetary policy. The second is a sharper decline in consumer demand, such that the zero-inflation steady state is no longer viable, absent a response of fiscal policy. In all exercises, we hold the discount rate ($\hat{\rho}$) of the entrepreneurs constant.

We explore the mild scenario in Figure 12. The figure depicts the region of the state space for which the ELB does not bind. The initial (and final) steady state is depicted by the intersection of the solid “ $\dot{c} = 0$ ” and “ $\dot{\pi} = 0$ ” lines. The dashed $\dot{c} = 0$ locus is drawn for $\rho = \underline{\rho}$, when we maintain the interest rate rule intercept constant at ι . From (26), a decline in ρ shifts the locus down in $\pi \times c$ space. The Phillips curve does not shift, as we maintain the $\hat{\rho}$ constant in this scenario.

The dashed arrows depict the dynamics that hold for $t \in [0, t')$; that is, relative to the dashed locus. Perfect foresight and worker and firm optimization imply that the equilibrium must be back at the initial steady state at $t = t'$. The purple line depicts trajectories that depart and return

to that steady state. The precise point on the trajectory that holds at $t = 0$ depends on t' .

As in the case of Figure 8, the potential trajectories correspond to combinations that clear the bond market. In this experiment, b^o is held fixed, but, all else equal, more patient workers desire to hold more bonds at a given real wage and interest rate. The market clears by a combination of lower c and higher \dot{c} , both of which reduced demand. The counterpart is a change in the path of π that ensures the worker consumption dynamics are consistent with firm optimization, which corresponds to lower π and higher $\dot{\pi}$. The inflation path in turn generates a path of the real interest rate via the interest rate rule. A temporary decline in the discount rate, therefore, generates the familiar temporary declines in inflation and consumption.

The monetary authority, nevertheless, could keep the equilibrium at the desired zero-inflation steady state by having \bar{i} move one-to-one with $\rho(t)$. In particular, a transitory decrease in the nominal interest rate that matches the decline in ρ maintains bond and good market equilibrium at the initial allocation. Specifically, let $\bar{i}(t)$ in the interest rate rule be adjusted as follows:

$$\bar{i}(t) = \rho(t) - \alpha + (\rho(t) + \lambda)(\lambda + \alpha)b^o/c^*. \quad (36)$$

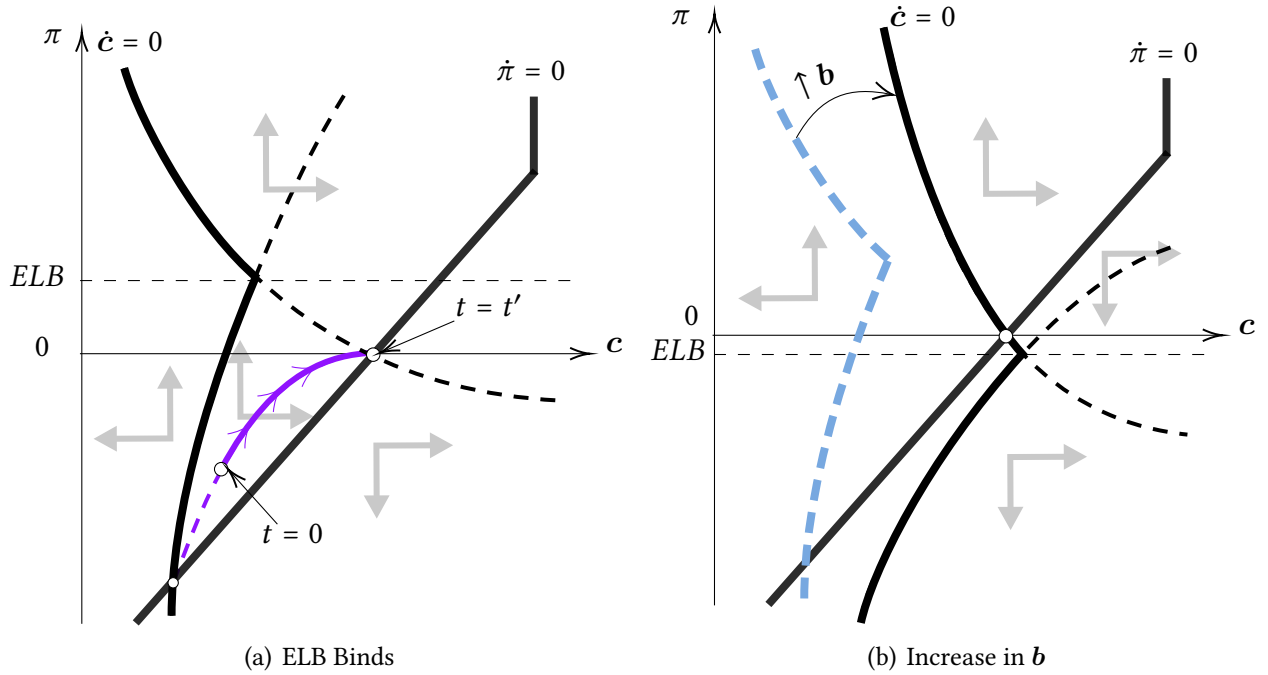
From (20), this modification ensures that the nominal interest adjusts so that at zero inflation the bond market clears at $c = c^*$. Hence, the economics of a moderate temporary decline in the discount rate track that of the textbook Ricardian model. Interestingly, it also echoes the dynamics of the anticipated deficit scenario above but shifted in $\pi \times c$ space relative to the initial steady state.

Perhaps a more interesting scenario is one in which the ELB prevents the monetary authority from fully accommodating the decline in ρ via a decline in the nominal interest rate. This will be the case if $\underline{\rho}$ is sufficiently low such that $\bar{i}(t)$ defined in (36) is negative when $\rho(t) = \underline{\rho}$. In this case, the ELB threshold is at positive inflation.

This case is depicted in Figure 13 panel (a). We omit the initial (and final) $\dot{c} = 0$ curve, but it intersects the $\dot{\pi} = 0$ at the horizontal axis with the ELB at negative inflation. The downward and upward sloping $\dot{c} = 0$ lines represent equations (26) and (35), respectively, with ρ set to $\underline{\rho}$. The “ELB” line is the inflation threshold at which $i = 0$ (i.e., $\pi = -\bar{i}/\theta_\pi$) under the time-varying interest rate rule with intercept (36) for $t \in [0, t')$, which is positive in this case of when $\rho = \underline{\rho}$.

The decline in ρ makes the zero inflation steady state not achievable for $t \in [0, t')$. The decline in consumer demand is so severe that it reduces interest rates below the ELB point. The dynamics for $t \in [0, t')$ are governed by the upward sloping “ $\dot{c} = 0$ ” locus, and the intersection of this curve with the $\dot{\pi} = 0$ Phillips curve is an unstable steady state (see footnote 33). The trajectory that leads away from this intersection and reaches the zero-inflation steady state at $t = t'$ is the

Figure 13: Temporary Decline in ρ : Binding ELB and Fiscal Policy



equilibrium.³⁵ This trajectory follows the logic set out in Werning (2011) in his “no-commitment” policy, and echoes that of Krugman (1998), Eggertsson and Woodford (2003), and Jung, Teranishi, and Watanabe (2005). There is initially a large decline in worker consumption and inflation, and the economy slowly recovers anticipating a return to “normal” at $t = t'$.

Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005), and Werning (2011) argue that commitment to low interest rates after the ELB no longer binds can improve upon the no-commitment policy. Commitment to this post- t' inflationary policy “pulls up” the trajectory: at time t' , the equilibrium has higher inflation and income than the zero-inflation steady state, and via anticipation, the entire trajectory adjusts. This logic underlies the now-standard “forward guidance” policy prescription when at the ELB.

In our environment, fiscal policy can provide an alternative solution and one that does not require the same type of commitment as forward guidance.³⁶ To see this suppose that the monetary policy rule features $\theta_b = \theta_b^*$, and the fiscal policy expands, $b' > b^0$. This shifts the upward-sloping portion of the “ $\dot{c} = 0$ ” locus down. For modest increases in b' , the intersection anchoring the trajectory in Figure 13 moves closer to the zero-inflation steady state. For large enough increases, the curve shifts enough that the ELB no longer binds, which occurs when the associated real rate

³⁵Here, we ignore the possibility that there are additional deflationary steady states, which may be possible given the (omitted) vertical portion of the Phillips curve.

³⁶Correia et al. (2013), Mian, Straub, and Sufi (2022), and Wolf (2021) also discuss dealing with the ELB using alternative fiscal policy tools.

at zero inflation is no longer negative. This case is shown in Figure 13 panel (b): fiscal policy is sufficiently large to render the ELB irrelevant, and the monetary authority can implement the target of $c = c^*$ and $\pi = 0$. Note again the need for monetary and fiscal policy coordination as after the fiscal expansion, the monetary authority must adjust its interest rate rule as the underlying permanent real rate at zero inflation has increased.

It could be the case that the ELB binds indefinitely in our non-Ricardian environment for a given set of parameters; that is, $r_*(b^0) < 0$. If the monetary authority targeted the zero-inflation steady state by setting $\iota = r_*(b^0)$, in Figure 13 the economy would be at the intersection of the $\dot{\pi} = 0$ line and the lower $\dot{c} = 0$, and would have permanent deflation and $c < c^*$. The monetary authority has the option to avoid this by raising the intercept of its interest rate rule, setting $\bar{\iota} > 0 > r_*(b^0)$. This would shift the upper $\dot{c} = 0$ locus out, and the ELB threshold down, creating the opportunity for a positive inflation steady state with $c > c^*$. At this steady state, $i > 0$ and the ELB would not bind, but inflation and worker consumption are higher than the original target of $c = c^*$ and $\pi = 0$. Fiscal policy provides an alternative path to the zero-inflation steady state. An increase in government debt can increase the long-run real interest rate to the point that the zero-inflation steady state is attainable with $i > 0$.

7 Conclusion

This paper investigated the interaction of fiscal and monetary policy in a non-Ricardian New Keynesian model. We demonstrated how increasing government bond supply can generate Pareto improvements, and characterized the mix of fiscal policy and monetary rule shifts needed to engineer them. We developed a phase diagram to characterize the global equilibrium dynamics. We used the diagram to analyze anticipated deficit effects and revisited how the global dynamics of the "forward premium puzzle" are sensitive to the knife-edge Ricardian equivalence case. We also explored using fiscal policy to escape a liquidity trap. We think that the insights from this tractable non-Ricardian framework provide lessons for more general heterogeneous agent models, including quantitative HANK models.

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A Proofs

Proof of Lemma 1

Proof. Given the log-log preferences, the inequality constraints will not bind at an optimum at almost all t , and hence we will ignore them in what follows.

Letting μ denote the (current value) co-state on assets, the Hamiltonian for the worker's problem is:

$$\begin{aligned} \mathcal{H}(s, t, c, n, \mu) = \\ \ln c(s, t) + \psi \ln(1 - l(s, t)) + \mu(s, t) (w(t)z(s, t)l(s, t) + (r(t) + \lambda)a(s, t) - c(s, t) - T(s, t)). \end{aligned}$$

The first-order conditions for c and l are:

$$\frac{1}{c(s, t)} = \mu(s, t) \quad (37)$$

$$\frac{\psi}{1 - l(s, t)} = w(t)z(s, t)\mu(s, t). \quad (38)$$

Eliminating μ by combining (37) and (38) generates (7).

The evolution of the co-state is given by (suppressing s and t)

$$\dot{\mu} = (\rho + \lambda)\mu - \frac{\partial \mathcal{H}}{\partial a} = (\rho - r)\mu.$$

From this and (37), we obtain the familiar Euler Equation (6).

We can integrate the Euler Equation forward to obtain:

$$\begin{aligned} \int_t^\infty R(t, \tau)c(s, \tau)d\tau &= c(s, t) \int_t^\infty R(t, \tau)e^{\int_t^\tau (r(m) - \rho)dm}d\tau \\ &= c(s, t) \int_t^\infty e^{-(\rho + \lambda)(\tau - t)}d\tau = \frac{c(s, t)}{\rho + \lambda}. \end{aligned} \quad (39)$$

Substituting this into the budget set (4), we obtain the “consumption function” that relates consumption at time t to financial assets and “human wealth,” net of taxes, (8). \square

Proof of Lemma 2

Proof. The static labor-consumption condition (7) can be integrated across cohorts to obtain (11). Aggregating the consumption function (8) gives (12).

Taking the time derivative of the aggregate consumption definition we have

$$\begin{aligned} \dot{C}^w(t) &= c(t, t)\dot{\phi}(t, t) + \int_{-\infty}^t \dot{c}(s, t)\phi(s, t)ds + \int_{-\infty}^t c(s, t)\dot{\phi}(s, t)ds \\ &= c(t, t)(\lambda + n)e^{nt} + \int_{-\infty}^t (r(t) - \rho)c(s, t)\phi(s, t)ds - \lambda \int_{-\infty}^t c(s, t)\phi(s, t)ds \\ &= (r(t) - \rho - \lambda)C^w(t) + (\lambda + n)c(t, t)e^{nt} \end{aligned}$$

where the second line uses the first order condition for household consumption and the evolution of ϕ .

Note that using $a(t, t) = 0$, we have

$$c(t, t) = \left(\frac{\rho + \lambda}{1 + \psi} \right) (h(t, t) - \mathcal{T}(t, t)).$$

Note as well that using the process for z , we have

$$h(s, t) = e^{-\alpha(t-s)} h(t, t)$$

and thus

$$H(t) = h(t, t) \int_{-\infty}^t e^{-\alpha(t-s)} \phi(s, t) ds = \frac{(\lambda + n)e^{nt}}{\alpha + \lambda + n} h(t, t)$$

Similarly we obtain

$$\mathcal{T}(t) = \frac{(\lambda + n)e^{nt}}{\alpha + \lambda + n} \mathcal{T}(t, t).$$

Thus,

$$\begin{aligned} (\lambda + n)e^{nt} c(t, t) &= (\lambda + n)e^{nt} \left(\frac{\rho + \lambda}{1 + \psi} \right) (h(t, t) - \mathcal{T}(t, t)) \\ &= \frac{\rho + \lambda}{1 + \psi} (\alpha + \lambda + n) (H(t) - \mathcal{T}(t)) \\ &= (\alpha + \lambda + n) C^w(t) - \frac{\rho + \lambda}{1 + \psi} (\alpha + \lambda + n) A(t). \end{aligned}$$

Plugging this back into the aggregate Euler equation, we have:

$$\begin{aligned} \dot{C}^w(t) &= (r(t) - \rho - \lambda)C^w(t) + (\lambda + n)c(t, t)e^{nt} \\ &= (r(t) - \rho + \alpha + n)C^w(t) - \frac{\rho + \lambda}{1 + \psi} (\alpha + \lambda + n) A(t). \end{aligned}$$

which delivers equation (10).

Using the definition of aggregate assets, and the first order condition for household labor, we get

$$\begin{aligned} \dot{A}(t) &= a(t, t)\phi(t, t) + \int_{-\infty}^t (\dot{a}(s, t)\phi(s, t) + a(s, t)\dot{\phi}(s, t))ds \\ &= r(t)A(t) + w(t)Z(t) - (1 + \psi)C^w(t) - \bar{T}(t) \\ &= (r(t) - \rho - \lambda)A(t) + w(t)Z(t) - \bar{T}(t) - (\rho + \lambda)(H(t) - \mathcal{T}(t)), \end{aligned}$$

and using (11) delivers (13), and completes the proof. □

Proof of Lemma 3

Proof. Given the CES aggregator across intermediates, we have demand for a good with price $p_j = p$ given by

$$y^D(p, t) = \left(\frac{p}{P} \right)^{-\eta} Y,$$

where $P = P(t)$ is the aggregate price index and $Y = Y(t)$ is final demand. Facing a real wage of $w(t)$, this implies

that real profits at time t are

$$\begin{aligned}\Pi(p, t) &= \frac{p}{P(t)} y^D(p, t) - w(t) y^D(p, t) \\ &= \left(\frac{p}{P(t)} - w(t) \right) \left(\frac{p}{P(t)} \right)^{-\eta} Y(t).\end{aligned}$$

The firm's Hamiltonian is given by:

$$\mathcal{H}(t, p, x, \mu) = \Pi(p, t) - f(x)Y(t) + \mu(t)x(t)p(t),$$

where we repurpose $\mu(t)$ to be the co-state on the price adjustment equation $\dot{p}(t) = x(t)p(t)$. The first-order condition with respect to $x(t)$ is:

$$\mathcal{H}_x = 0 \Rightarrow f'(x)Y(t) = \mu(t)p(t)$$

Imposing symmetry across all firms, we have that $p(t) = P(t)$ and $x(t) = \dot{P}(t)/P(t) \equiv \pi(t)$. Then,

$$f'(\pi)Y(t) = \mu(t)P(t).$$

The first-order condition with respect to the state $p(t)$ is:

$$\begin{aligned}\dot{\mu}(t) &= \hat{\rho}\mu(t) - \mathcal{H}_p(t, p(t), x(t), \mu(t)) \\ &= \hat{\rho}\mu(t) - \left(\frac{p}{P(t)} \right)^{-\eta} Y(t) \left[\frac{1}{P(t)} - \eta \frac{1}{p} \left(\frac{p}{P(t)} - w(t) \right) \right] - \mu(t)x(t) \\ &= \hat{\rho}\mu(t) - \frac{Y(t)}{P(t)} (1 - \eta + \eta w(t)) - \mu(t)\pi(t),\end{aligned}$$

where the last line imposes symmetry.

Differentiating the condition $\mathcal{H}_x = 0$ with respect to time and substituting we get

$$\frac{\dot{\pi}(t)}{\pi(t)} + \frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{P}(t)}{P(t)} + \frac{\dot{\mu}(t)}{\mu(t)}.$$

Solving for $\dot{\pi}$ and using the previous equation for $\dot{\mu}$, we get:

$$\dot{\pi} = (\hat{\rho} - g_Y)\pi + \kappa [w^* - w] \quad \text{if } \pi \in [\underline{\pi}, \bar{\pi}]$$

For $\pi \notin [\underline{\pi}, \bar{\pi}]$, we have:

$$\begin{aligned}(\hat{\rho} - g_Y)\underline{\pi} &= \kappa (w - w^*) && \text{if } \pi < \underline{\pi} \\ (\hat{\rho} - g_Y)\bar{\pi} &= \kappa (w - w^*) && \text{if } \pi > \bar{\pi}.\end{aligned}$$

□

Proof of Lemma 4

Proof. The only missing element in the derivation of Lemma 4 is to establish that no cohort has a negative asset position in the original BGP. Given the premise of a stationary equilibrium, we drop time arguments when possible. The budget constraint for cohort s at time t is given by equation (3):

$$\dot{a}(s, t) = (r + \lambda)a(s, t) + wz(s, t)l(s, t) - c(s, t) - T(s, t).$$

We have from (7),

$$wz(s, t)l(s, t) = wz(s, t) - \psi c(s, t),$$

and from (2) $T(s, t) = z(s, t)T(t)$ where $T(t) = T$, given that we are in a BGP. Finally, from (8), we have

$$\begin{aligned} c(s, t) &= \left(\frac{\rho + \lambda}{1 + \psi} \right) (a(s, t) + h(s, t) - \mathcal{T}(s, t)) \\ &= \left(\frac{\rho + \lambda}{1 + \psi} \right) \left(a(s, t) + (w - T) \int_t^\infty R(t, \tau)z(s, \tau)d\tau \right), \end{aligned}$$

where the second line uses the definition of h and \mathcal{T} . Also,

$$\int_t^\infty R(t, \tau)z(s, \tau)d\tau = \frac{z(s, t)}{r + \alpha + \lambda - g}$$

which requires $r + \alpha + \lambda - g > 0$ for bounded human wealth. Substituting the consumption into the \dot{a} equation above, and rearranging, we have:

$$\dot{a}(s, t) = (r - \rho)a(s, t) + z(s, t)(w - T) \left(\frac{r - \rho - g + \alpha}{r + \alpha + \lambda - g} \right)$$

In a BGP with $b^o \geq 0$, (20) implies $r \geq \rho + g - \alpha$. Non-negative consumption and zero assets at birth imply $w - T \geq 0$. Thus, the last term is positive. Starting from zero assets at birth, this indicates assets can never go below zero, as $\dot{a} \geq 0$ when $a = 0$, which completes the proof. \square

Proof of Lemma 6

Proof. Utility of cohort s at time t is given by:

$$U(s, t) = \int_t^\infty e^{-(\rho + \lambda)(\tau - t)} [\log c(s, \tau) + \psi \log(1 - l(s, \tau))] d\tau.$$

In a BGP, the Euler Equation implies $c(s, \tau) = c(s, t)e^{(r - \rho)(\tau - t)}$, and the static first-order condition

$$1 - l(s, \tau) = \frac{\psi c(s, \tau)}{wz(s, \tau)} = \frac{\psi c(s, t)e^{(r - \rho)(\tau - t)}}{wz(s, \tau)}.$$

Substituting in, we have

$$\frac{\rho + \lambda}{1 + \psi} U(s, t) = \left(\log c(s, t) + \frac{r - \rho}{\rho + \lambda} - \frac{\psi \log w}{1 + \psi} \right) - \zeta_0(s, t)$$

where

$$\begin{aligned}\zeta_0(s, t) &\equiv \psi \int_t^\infty e^{-(\rho+\lambda)(\tau-t)} \log z(s, \tau) d\tau \\ &= \frac{\psi(g-\alpha)}{\rho+\lambda} t + \frac{\psi\alpha}{\rho+\lambda} s + \frac{\psi(g-\alpha)}{(\rho+\lambda)^2} + \frac{\psi \log z_0}{\rho+\lambda}.\end{aligned}$$

In a BGP,

$$\begin{aligned}h(s, t) &= \frac{z_0 w e^{gt-\alpha(t-s)}}{r+\lambda+\alpha-g} \\ \mathcal{T}(s, t) &= \frac{z_0 T e^{-\alpha(t-s)}}{r+\lambda+\alpha-g},\end{aligned}$$

This implies

$$(1+\psi)c(s, t) = (\rho+\lambda) \left(a(s, t) + \frac{z_0(w-T)e^{gt-\alpha(t-s)}}{r+\lambda+\alpha-g} \right).$$

Substituting into the expression for $U(s, t)$ we obtain

$$\left(\frac{\rho+\lambda}{1+\psi} \right) U(s, t) = \log \left(a(s, t) + \frac{z_0(w-T)e^{gt-\alpha(t-s)}}{r+\lambda+\alpha-g} \right) + \frac{r-\rho}{\rho+\lambda} - \frac{\psi \log w}{1+\psi} + \log \left(\frac{\rho+\lambda}{1+\psi} \right) - \frac{\rho+\lambda}{1+\psi} \zeta_0(s, t),$$

Using that $w = (1+\psi)c$ and that $T = (r-g-n)b$ together with (20) we have

$$\left(\frac{\rho+\lambda}{1+\psi} \right) U(s, t) = \log \left(\frac{a(s, t)}{w} + \frac{\rho+\lambda+g+n-r}{(\alpha+\lambda+n)(\rho+\lambda)} z_0 e^{gt-\alpha(t-s)} \right) + \frac{r-\rho}{\rho+\lambda} + \frac{\log w}{1+\psi} + \log \left(\frac{\rho+\lambda}{1+\psi} \right) - \frac{\rho+\lambda}{1+\psi} \zeta_0(s, t).$$

Using the value of $\zeta_0(s, t)$, we obtain after some manipulation:

$$\begin{aligned}\left(\frac{\rho+\lambda}{1+\psi} \right) U(s, t) &= \log \left(\frac{a(s, t)(\rho+\lambda)(\alpha+\lambda+n)}{w z_0 e^{gt}} + (\rho+\lambda+g+n-r) z_0 e^{-\alpha(t-s)} \right) + \\ &\quad + \frac{r-\rho}{\rho+\lambda} + \frac{\log w}{1+\psi} + \frac{1}{1+\psi} gt + \frac{\psi}{1+\psi} \alpha(t-s) + u_0\end{aligned}$$

where

$$u_0 \equiv -\log((\alpha+\lambda+n)(1+\psi)) - \frac{\psi}{1+\psi} \frac{g-\alpha}{\rho+\lambda} + \frac{1}{1+\psi} \log z_0$$

Evaluating at $s = t$, we obtain:

$$\left(\frac{\rho+\lambda}{1+\psi} \right) U(t, t) = \log(\rho+\lambda+g+n-r) + \frac{\log w}{1+\psi} + \frac{r-\rho}{\rho+\lambda} + \frac{1}{1+\psi} gt + u_0$$

□